Parameterized Charge Deposition Model for Double-Sided Silicon

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Abstract

We discuss a model for simulating charge on double-sided silicon detectors based on a parameterization of the underlying physical process: primary ionization, $\delta$-rays, charge diffusion, capacitive charge sharing between strips, and electronic noise.

1 Introduction

A fast and faithful simulation of charge on strips is required for development of tracking algorithms as well as run 2 physics. Obviously, the model used for run 1 (SVX') did not simulate double-sided detectors. [1] The strategy there was to directly model clusters from the data, folding in various effects (e.g. $\delta$-rays, noise) using distributions directly derived from the data. Here we have chosen an alternative approach based on a piece-by-piece inclusion of known effects for the following reasons: 1) lack of data; 2) the desire to faithfully reproduce correlations on both sides of the detector, including $\delta$-rays; and 3) an attempt to produce a model that can be easily re-tuned to the data as the detectors age.

Cluster distributions in the existing SVX II testbeam data have been studied previously.[2] Unfortunately, this data exists only for normally incident tracks. Furthermore, this data obviously does not include the effects of the magnetic field. We were thus motivated to develop a model based on a GEANT simulation of the ionization including $\delta$-rays. We have separately added a charge diffusion model based on standard formulae, capacitive charge sharing, and noise as described below. The level of charge sharing and noise are chosen to be in agreement with the test beam data. They remain as tuneable parameters in the model.

Using this GEANT model we have extracted distributions to parameterize the diffusion and $\delta$-rays. Charge sharing and noise is added exactly as in the GEANT model. No magnetic field is used at present. The description of this parameterization is the purpose of this note.

There are two levels of testing that remain to be done. First, a comparison of the parameterized model to the GEANT model. A first attempt at this is made
here. Second, a comparison with the previously analyzed test beam data. Following this, the effect of the magnetic field has to be added to the GEANT model, and the parameterization suitably revised.

We foresee the parallel development of a parameterization along the lines of the SVX' simulation. A model that gets the correlations on the two sides basically correct should not be difficult to develop. It is not obvious to us at this point that the details of the correlations will ever be observable in the data, or which approach will lead to a faster, better and more easily tuned model.

2 GEANT Model

The GEANT simulation code provides full particle transport and simulation of interactions, including the amount of energy lost. A small fraction of particles passing through the silicon will produce high-energy δ-rays. These can travel a significant distance in the silicon and increase the cluster width. GEANT explicitly generates these δ-rays and transports them. We have put into the simulation a calculation of the charge drift and diffusion. A fraction of charge is lost to the neighboring strips due to the capacitive coupling to the neighbors. This is included and noise is also added. An example of an event is shown in figure 1. Figure 2 shows the cluster width distributions for both the \( r\phi \) and \( z \) sides and the number of 1, 2 and 3 strip cluster events as a function of the interaction position.

2.1 Simulation Details

For the purpose of calculating the charge deposited on the electrode the step size is limited to a maximum of 5 \( \mu m \). The point midway in the step is used as the point from which the charge originates. This charge is spread out via diffusion as described below.

In GEANT it is possible to explicitly generate δ-rays as a separate particle which is then tracked. It also possible to have the effects of δ-rays included in the energy loss calculation without explicitly tracking the δ-rays. Even though both methods give the same total energy loss, it is important to explicitly generate the δ-rays as these δ-rays can travel a significant distance and hence increase the cluster width. Figure 1 shows an event where an energetic δ-ray is produced.

2.2 Drift and Diffusion

As energy is deposited electron hole pairs are created at a rate of about one \( e/h \) pairs per 3.6 eV of deposited energy. Electrons drift to the n-side and holes drift to the p-side.
Figure 1: An example of a GEANT event with a high-energy $\delta$-ray. The pitch is 60 $\mu$m.
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**Figure 2:** Cluster distributions on $r\phi$ and $z$ sides and number of 1, 2 and 3 strip clusters as a function of interaction point. The middle plots use the actual interaction point and the bottom plots have include smearing of the interaction point to take into account the positional resolution.
The drift velocity is given by

\[ v = \pm \mu_{p/n} E, \]  

(1)

where \( \mu_{p/n} \) are the hole/electron mobilities. For an abrupt junction the electric field is given by,

\[ E(x) = E_0 + E_1 x \]  

(2)

\[ E_0 = -(V_{app} + V_{dep})/d \]  

(3)

\[ E_1 = 2V_{dep}/d^2 \]  

(4)

As the electrons and holes drift to the electrodes they diffuse with a Gaussian spatial distribution with sigma of

\[ \sigma = \sqrt{2Dt} \]  

(5)

where \( D \) is the diffusivity and is given by

\[ D = k_B T \mu /q \]  

(6)

where \( k_B \) is Boltzmann’s constant, \( T \) is the temperature in Kelvin and \( q \) is the electron charge.

The time is a function of the distance to the electrode and the electric field. The resulting charge distribution will be a superposition of several Guassians.

The time to drift to the electrode is given by

\[ t = \int \frac{dx}{v} = \int \frac{dx}{\mu E} \]  

(7)

\[ = \pm \frac{1}{\mu E_1} \ln \left( \frac{E_0 + E_1 x_f}{E_0 + E_1 x_i} \right) \]  

(8)

In the equation for \( \sigma \) the mobilities exactly cancel and so the diffusion is independent of the mobility.

### 2.3 Effects of magnetic field on drift and diffusion

The magnetic field causes the electrons and holes to drift at an angle (the Lorentz angle). The Lorentz angle \( \omega \) is given by \( \tan(\omega) = \mu B \). Typical values for the mobility are 0.05 and 0.13 \( m^2/V/s \) for holes and electrons respectively. Since the magnetic field is in the same direction as the strips on the z-side the drift angle is only of relevance on the r-phi side.

As well as the drift angle the magnetic field causes the carriers to take slightly longer to get to the surface and consequently increase the diffusion slightly. The increase in diffusion time is given by \( 1/\cos(\omega) \).
2.4 Charge sharing

A fraction of charge is lost to the neighbors due to capacitive coupling. The AC coupling capacitor $C_{AC}$ and the inter-strip capacitance $C_1$ act as a charge divider. The fraction of charge loss is given by

$$\frac{C_1}{(C_{AC} + C_1)}$$  \hfill (9)

The charge gained by the neighbors is half this value for each side. This charge is put on the closest neighbors. The charge gained on the next to nearest neighbors is ignored.

2.5 Noise

Noise is added by generating a random number with Gaussian distribution with sigma given by the RMS noise. Noise is added to the cluster strips plus three strips on either side of the cluster. The values of the RMS noise, 1540 e and 1600 e for X and Z sides respectively, are taken from testbeam data.  [3]

2.6 Spatial smearing

The simulation naturally has the exact intersection position. However, for comparison with data this position is smeared by the resolution. These resolutions have been assumed to be 10% of the width.

2.7 Distributions from GEANT Model

Figures 3-6 show the X-strip (p-side) and Z-strip (n-side) deposited charge as simulated in GEANT. Figure 3 is the total charge deposited on the strips including noise. Figure 4 shows only the ionization charge from the primary track and δ-rays. Here the X and Z side charge is one-hundred percent correlated. Figures 5 and 6 show that the ionization charge from the primary track has Gaussian fluctuations, whereas the δ-rays contribute the Landau tail.
Figure 3: Total ionization charge deposited on $r - \phi$ (x) and Z sides. Fit is to a Landau convoluted with a gaussian.
Figure 4: Charge deposited on $r - \phi$ (x) and Z sides including noise. Fit is to a Landau convoluted with a gaussian.
Figure 5: Ionization charge in events with no δ-rays.
Figure 6: Ionization charge in events with at least one $\delta$-ray.
3 Parameterization

We now describe the parameterization extracted from the GEANT-based model. This includes: ionization from the primary track, diffusion, and δ-rays. Charge sharing and noise are added just as in the GEANT-based model.

3.1 Primary charge

The charge deposited by the primary particle is taken from the restricted energy loss formula, which includes collision losses up to some maximum value $T_{cut}$. [4] The Bethe-Block curve is recovered by setting $T_{cut} = T_{max}$, where $T_{max}$ is the kinematic maximum energy loss per collision. We have chosen a value of $T_{cut} = 10$ KeV to match the minimum δ-ray kinetic energy produced by GEANT. This value should probably be increased significantly to increase the efficiency of the simulation. Figure 7 shows the restricted energy loss versus $\beta\gamma$ curve for 300 $\mu$m thick silicon. For reference, the Bethe-Block curve is also shown.

3.2 Diffusion

Charge diffusion is modeled as a charge sharing fraction as a function of the track position (figure 8). The effect on the strip charge is shown in figures 9 and 10. For the normally incident track near a strip boundary, the charge sharing due to diffusion has a significant effect. On the other hand, for tracks at large angles of incidence the diffusion is negligible because charge diffuses in both directions across a strip boundary. This justifies the use of a single diffusion profile derived from normally incident tracks.

3.3 δ-rays

The δ-ray multiplicity distribution derived from GEANT is shown in figure 11. The energy of the δ-ray is then chosen from distributions shown in figure 12. We have extracted the range-energy relationship for δ-rays from GEANT. This function is plotted in figure 13. The ionization loss of the δ-ray is then parameterized by its mean lateral range, where the lateral direction is perpendicular to the direction of the track and approximately in the plane of the silicon. A fraction of the δ-rays will exit the silicon according to their lateral range distribution in figure 14. Next, range fluctuations in the plane of the silicon are taken from approximately Gaussian distributions according to the mean lateral range of the δ-ray (figure 15). Finally, the ionization charge from the δ-ray is deposited on the strips according to ionization loss profiles (figure 16). These profiles show the characteristic Bragg peak around the mean range. The profiles vary somewhat as a function of the mean range, so we have chosen profiles in nine bins of range. Note that the mean range is defined as the δ-ray end-point, so that there is on average some ionization loss beyond the Bragg peak.
due to the possibility of the $\delta$-ray scattering back (in the lateral plane) towards the primary track. This ionization charge is then deposited appropriately on the strips according to the diffusion model.

3.4 Distributions from parameterized model

We have made comparisons of the parameterized model and the original GEANT simulation for 1 GeV pions. Figures 17-20 compare the parameterization to GEANT for normally incident tracks. We see that the original distributions are nicely reproduced. There is however a problem with the X-Z charge correlation in the $\delta$-rays which remains to be fixed. Figure 21 compares the charge distributions for tracks with a range of incident angles. Again, the GEANT distributions are nicely reproduced.

4 Outlook

We have shown that a simulation of charge deposition in silicon which separately models primary ionization, charge diffusion, capacitive sharing, and noise can be well modeled by a relatively simple parameterization. A number of studies of the parameterized model remain to be done. Most notably missing from the above study was any comparison of cluster-size distributions. In addition, it would be nice to compare the simulation directly with data. Next, the B-field must be included in GEANT and the parameterization suitably modified. Finally there is a concern for performance (CPU time required to generate hits).

References


Figure 7: Restricted energy loss for $T_{cut} = 10$ KeV (line) in 300 $\mu$m silicon. For comparison, the Bethe-Block formula is shown (dots).
Figure 8: Fraction of charge deposited on strip as a function of track position. Difference from unity is charge shared with the neighboring strip.
Figure 9: Charge deposition from normal track incident on the silicon near the edge of a strip. Top: path of track through silicon; bottom: charge on strip with (solid) and without (dotted) diffusion model.
Figure 10: Charge deposition from large-angle track incident on the silicon. Top: path of track through silicon; bottom: charge on strip with (solid) and without (dotted) diffusion model.
Figure 11: $\delta$-ray multiplicity per incident track.
Figure 12: $\delta$-ray energy distribution from GEANT (line) and as reproduced in the simulation (points).
exp fit to bins 70-100

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Figure 13: Relation between \(\delta\)-ray energy and mean range in GEANT.
Figure 14: Probability for $\delta$-ray to exit silicon as a function of its lateral range.
Figure 15: Lateral range fluctuations in bins of mean lateral range. The fluctuations are approximately Gaussian.
Figure 16: Ionization loss profiles in nine bins of mean range. The profile takes into account the rapid energy loss near the δ-ray’s end-point (Bragg peak).
Figure 17: Charge deposited on $r - \phi(x)$ and $Z$ sides, including noise, calculated with parameterization (points). Overlayed (line) is GEANT simulation.
Figure 18: Total ionization charge deposited on $r - \phi (x)$ and $Z$ sides calculated with parameterization (points). Overlayed (line) is GEANT simulation.
Figure 19: Ionization charge for events with no δ-rays.
SVXMC2 SIMULATION

Figure 20: Ionization charge for events with at least one $\delta$-ray. Overlayed (line) is GEANT simulation.
Figure 21: Comparison of parameterization and GEANT for tracks with a range of angles of incidence: $|\phi| < 5^\circ$ and $|\theta| < 15^\circ$. 