# Tracking for position and vertex.

## I Position measurements

- A record of the image of the track a particle makes when passing through material.
- Why? :

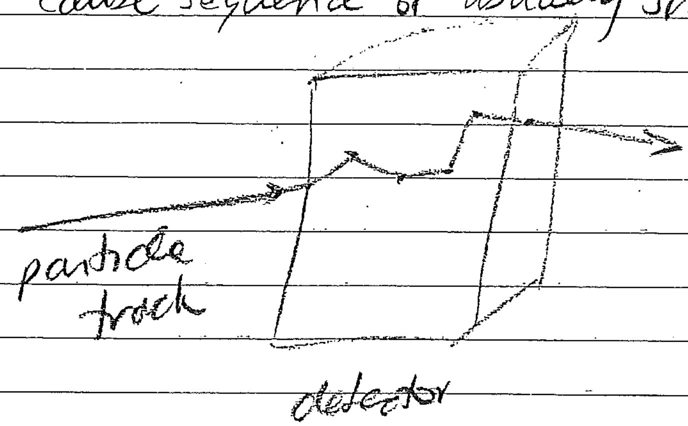
  i) for a short lived particle travelling at known velocity, (usually ~c) image of full track tells us its lifetime.

  ii) track points back to location of creation ("vertex", "source")

  iii) If vertex and expected location of primary production do not coincide, infer the existence of an undetected intermediate particle

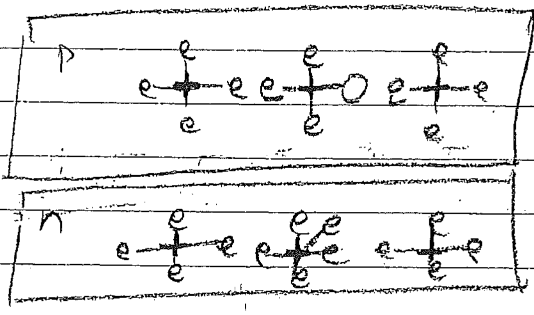
° Restriction: typically position is measured before anything else, because any detector material that precedes it will affect the particle + reduce the precision of the measurement. Material affect occurs as "multiple Coulomb scattering":

interactions with nuclei of detecting material cause sequence of usually small-angle scatters




Principle Technique:

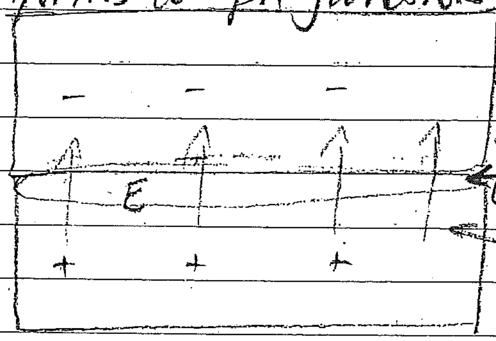
° Semiconductor detectors (usually Silicon)




neutral p-type semiconductor, lattice defects are missing $e^-$'s (ie they have effectively positive holes)

neutral n-type semiconductor, lattice defects have extra $e^-$'s.

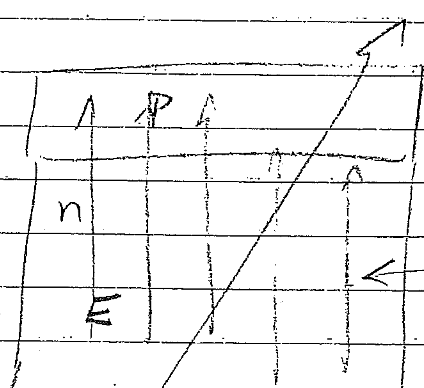
some carriers diffuse across the interface, neutralizing that region
This creates an $\vec{E}$ across the region, inhibiting further diffusion
This forms a pn junction




Excess of $e^-$ here means
{region depleted of charge}
additional $e^-$ don't want to cross

Typically this contact potential is ~1V.

Now apply more voltage across junction, sweep all the free $e^-$ from n-type region



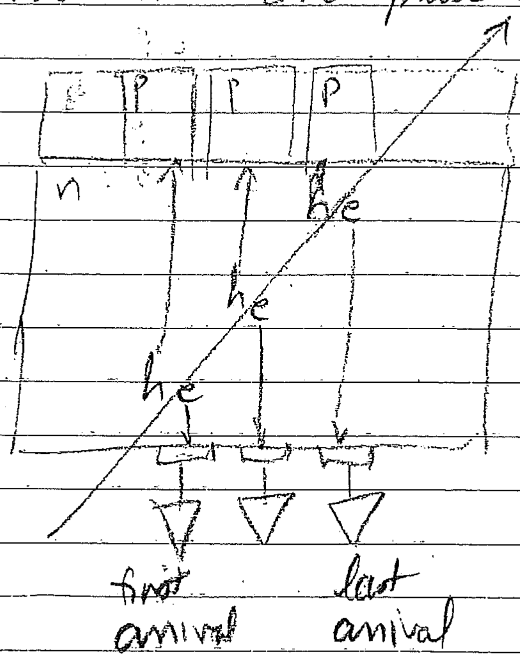

← no carriers in conduction band

now let a charged particle pass through device.

Charge excites valence e into conduction band, creating e-hole pairs

e and h separate, following $\vec{E}$ lines to opposite sides of semiconductor.

apply segmented electrodes to 1 or both sides, + record arrival time and pulseheight information




reconstruct track to few microns

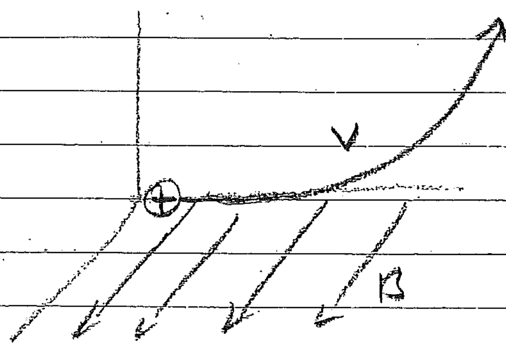
Semiconductors are typically configured as

- Charge-coupled devices (CCD's)
- Pixels
- Strip detectors

II. Momentum measurement

Principle:

A charged particle's path will curve in a magnetic field, because the particle feels $\vec{F} = q\vec{v} \times \vec{B}$




where $F_{centripetal} = \frac{mv^2}{r}$

Treat this as a centripetal acceleration, then the path describes the equilibrium situation $\hat{F}_{centripetal} = \hat{F}_{Lorentz}$

$$\frac{mv^2}{r} = qvB$$

$$p = mv = rqB$$

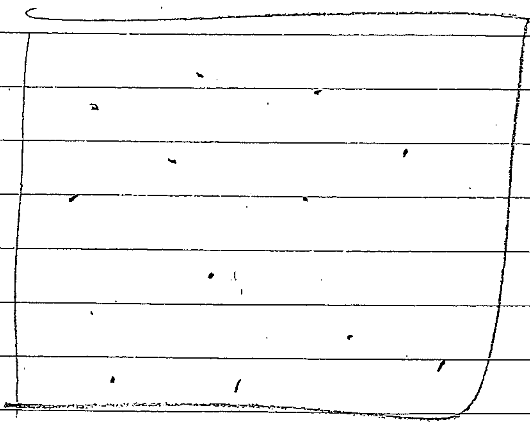
So if we:

- apply a known $\vec{B}$ to the region where particles are travelling
- guess $q = \pm 1$
- image the track → reconstruct $r$

Then we will know the particle's momentum $p$

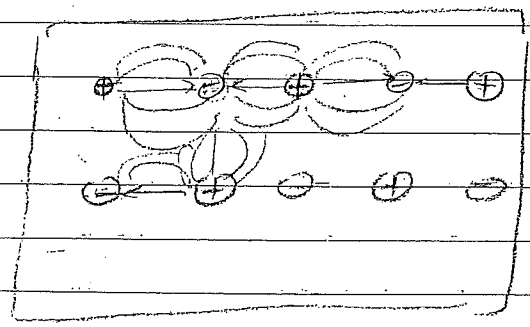
[If we subsequently measure $v$, we can infer $m$ → particle ID]

How to image the track:

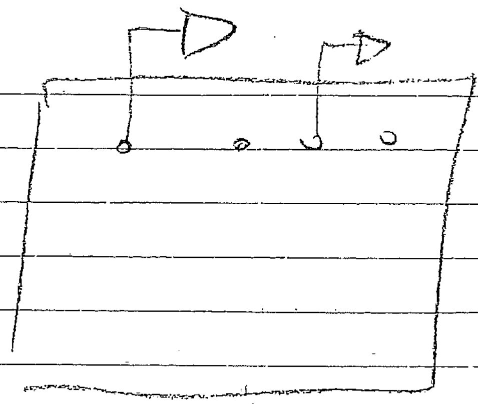


i) Fill the region with gas

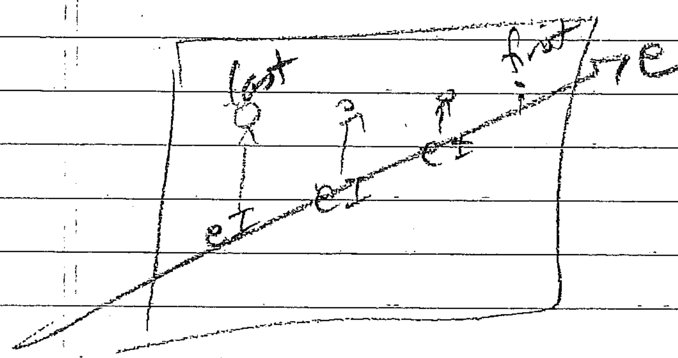
ii) Stretch wires across it, and apply positive or negative voltages to adjacent wires to fill the region with a well-defined $\vec{E}$ field



When a charged particle traverses this region it, ionizes gas molecules close to its track producing a trail of electrons and positive ions. Those e and I drift along the $\vec{E}$ lines to the wires, where they induce a current in the wires.



Connect electronics to the wires; record signals' pulseheight and time




Reconstruct track.

Infer radius of curvature's

Various species of this ionization technology:

One wire: "Proportional chamber"

Many wire: "Multiwire proportional chambers" MWP and "drift chambers"

Primary topics from Ch 8 to be covered:
1) R
2) Form factors
3) Feynman Rules for QCD

• R

Recall $\sigma = \int \frac{d\sigma}{d\Omega} d\Omega \sim e^2$ for tree level QED processes

Similarly for QCD processes

$\sigma \sim (Qe)^2$

quark charge ($^{+2}/_3$, $^{-1}/_3$, etc)

Inside $\sigma$ is $|M|^2$ so this is really $\sum_{i=\text{all quarks flavors involved in process}} (Qe)^2$

Consider $R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

only 1 type of electron so this contributes $e^2$

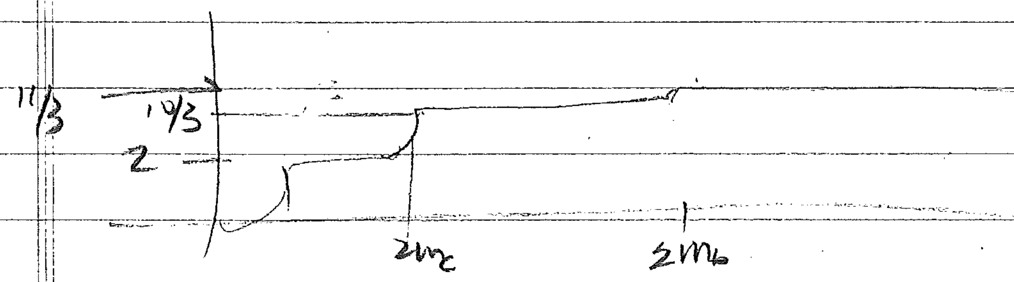
$\to 3 \sum_{i=1}^{n_{\text{flavors}}} Q_i^2 e^2$

# of flavors that can be produced depends on c-o-m energy of collision

each flavor produced in 3 colors

Gell-Mann: "Whatever is not forbidden is compulsory"

To discover new quark types, collide $e^+e^-$, scanning in com energy. Measure R. No need to find resonances, just count outgoing hadrons normalized to outgoing muons.




$$R(u,d,s) = 3\left[\left(\tfrac{2}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2\right] = 3\left[\tfrac{6}{9}\right] = 2$$

$$R(u,d,s,c) = \quad \cdots \quad + \left(\tfrac{2}{3}\right)^2 = 3\left[\tfrac{10}{9}\right] = \tfrac{10}{3}$$

$$R(u,d,s,c,b) \quad \cdots \quad + \left(\tfrac{1}{3}\right)^2 = 3\left[\tfrac{11}{9}\right] = \tfrac{11}{3}$$

etc.

This is the evidence for color.

# I. Form Factors + Structure Functions

To find out the structure of a complex object, we scatter probes from it and reconstruct information from their scattering pattern. The probes are typically $e^-$ although $\nu$ are also used.

4.5

The Rutherford formula describes the cross-section for non-relativistic collisions of 2 charged but spinless pointlike objects:

$$\left.\frac{d\sigma}{d\Omega}\right) = \frac{(q_1 q_2)^2}{16E^2 \sin^4\left(\frac{\theta}{2}\right)}$$

More generally the Rutherford formula includes relativistic terms (electric Sept. 19)

This must be modified if the target (proton) is

i) not spinless, and has $\vec{\mu}$

ii) not infinitely massive → target recoils

iii) not a point; charge distributed as $e\rho(x)$

Generically we say

$$\left.\frac{d\sigma}{d\Omega}\right|_{true} = \left.\frac{d\sigma}{d\Omega}\right|_{pointlike} \cdot |F(q)|^2$$

function describes the properties of a particular interaction w/o including all the underlying physics

Form factor depends on momentum transfer $q$ between probe and target (higher $q$ sees more details

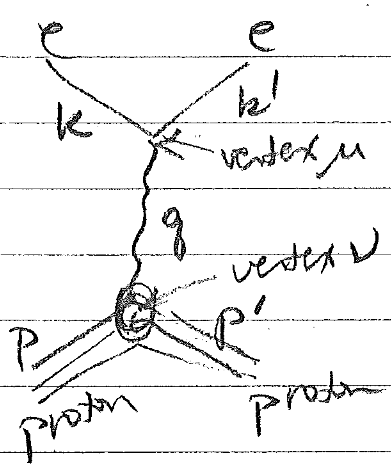
Begin with

$$\left.\frac{d\sigma}{d\Omega}\right|_{Mott} = \left(\frac{e^2}{4\pi}\right)^2 \frac{1}{\left(2\vec{p}^{\,2}\sin^2\frac{\theta}{2}\right)^2} \overbrace{\left[m^2 + \vec{p}^{\,2}\cos^2\frac{\theta}{2}\right]}^{\mathcal{M}_{Mott}}$$

Recall this came from

$$\mathcal{M} = -e^2\left[\bar{u}(k')\gamma^\mu u(k)\right]\frac{g_{\mu\nu}}{q^2}\left[\bar{u}(p')\gamma^\nu u(p)\right]$$

for



Recall bilinears are $\bar{\psi} I \psi$

$\bar{\psi}\gamma^{\mu}\psi$

$\bar{\psi}\gamma^{5}\psi$ x

$\bar{\psi}\gamma^{5}\gamma^{\mu}\psi$ x

$\bar{\psi}\sigma^{\mu\nu}\psi$

Henley & O'Neill p.94

ii) To add proton structure, replace $\gamma^{\nu}$ with some function "$\Gamma$" which is some linear combination of the bilinear covariants.

Experimentally, the EM interaction conserves parity, so leave out terms with $\gamma^5$.

Then in general

$$j_{\mu}(p,p') = \bar{u}(p')\Gamma_{\mu}u(p) = \bar{u}(p')\Big[\gamma_{\mu}K_1(q^2) + i\sigma_{\mu\nu}\underbrace{(p'-p)^{\nu}}_{q^{\nu}}K_2(q^2)$$

$$+ i\sigma_{\mu\nu}(p'+p)^{\nu}K_3(q^2) + \underbrace{(p'-p)_{\mu}}_{q_{\mu}}K_4(q^2) + (p'+p)_{\mu}K_5(q^2)\Big]u(p)$$

The K's are not yet determined.

The $(p'+p)$ terms are not really independent.

Because $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu},\gamma^{\nu}]$, and $p'-p = q$, they combine with the other terms. We are left with

anomalous magnetic moment $\frac{\alpha}{2\pi}$

$$j_{\mu}(p,p') = \bar{u}(p')\left[\gamma_{\mu}F_1(q^2) + i\left(\frac{K F_2(q^2)}{2M}\right)\sigma_{\mu\nu}q^{\nu} + q_{\mu}F_3(q^2)\right]u(p)$$

$M$ = proton mass

The $\frac{K}{2M}$ is a convention

Recall the electron $j_\mu$ is a current

Require current conservation : $j^\mu{}_{,\mu} = 0$

$\downarrow$

$$q^\mu j_\mu = 0$$

Then

$$\bar{u}(p')\left[\not{q} F_1 + \frac{iKF_2}{2M} q^\mu \sigma_{\mu\nu} q^\nu + q^2 F_3\right] u(p) = 0$$

Roe p. 201 Since each term is independent, each must individually satisfy current conservation. Check them:

Term #

1 $\quad \not{q} F_1 = (\not{p}' - \not{p}) F_1$

Apply Dirac Eq: $(\not{p} - m) u = 0$

$\downarrow$

$$\not{p} u = m u$$

$$(\not{p}' - \not{p}) F_1 u = \underbrace{(m-m)}_{0} F_1 u = 0$$

This satisfies $q^\mu j_\mu = 0$ so is ok

2. $\quad \sigma_{\mu\nu} q^\mu q^\nu = 0$

antisymmetric $\quad$ symmetric

Satisfies $q^\mu j_\mu$ so is ok

3 $\quad q^2 \neq 0$. Does not satisfy $q^\mu j_\mu$ so $F_3$ must $= 0$.

So we are left with

$$\boxed{j_\mu = \bar{u}(p')\left[\gamma_\mu F_1 + \frac{iK}{2M}\sigma_{\mu\nu}q^\nu F_2\right]u(p)}$$ as the most

general $e^-$ current that conserves charge + parity

Other conditions:

1) The proton will show no structure as the probe momentum goes to 0, so $F_1(q^2=0)=1$

2) By convention, set $F_2(q^2=0)=1$. Then the second term @ $q^2=0$ is $\frac{iK}{2m}\sigma_{\mu\nu}q^\nu$ = the anomalous $\vec{\mu}$ of the proton

(same form as for $e^-$ but with $K$) which is measured not derived.)

Plug this $j^\mu$ into $\mathcal{M}$, define $\tau \equiv \frac{-q^2}{4M^2}$, then

$$\boxed{\begin{aligned}\frac{d\sigma}{d\Omega} &= \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left\{\left[F_1^2 + \tau K^2 F_2^2\right] - \left[2\tau(F_1 + KF_2)^2 \tan^2\frac{\theta}{2}\right]\right\} \\ &= \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left[\left(F_1^2 + \tau K^2 F_2^2\right)\cos^2\frac{\theta}{2} - 2\tau(F_1+KF_2)^2\sin^2\frac{\theta}{2}\right]\end{aligned}}$$

The Rosenbluth Formula

Note

This formula does not depend for its structure on the charge of the proton. So it also applies generally to the neutron.

In these cases

$$F_1(q^2=0) \rightarrow \begin{cases} F_1^{proton}(0) = 1 \\ F_1^{neutron}(0) = 0 \end{cases}$$

$$F_2(q^2=0) \rightarrow \begin{cases} F_2^{proton}(0) = 1 \\ F_2^{neutron}(0) = 1 \end{cases}$$

$K^{proton} \cong +1.79$

$K^{neutron} \cong -1.91$

It is sometimes convenient to write the Rosenbluth formula without cross terms in $F_1 \cdot F_2$.

Define $G_E \equiv F_1 + \frac{Kq^2}{4M^2} F_2$ "Electric form factor"

$G_M \equiv F_1 + KF_2$ "Magnetic " "

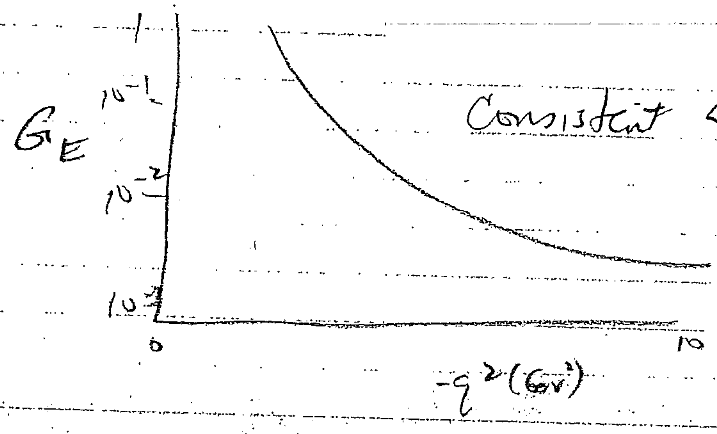
Then

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left\{ \frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2} \right]$$

$$= \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \frac{E'}{E} \left\{ \frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2} \right\}$$

$G_E$ and $G_M$ are generalizations of $F(q)$

We will interpret them as the charge ($G_E$) and magnetic moment ($G_M$) distributions:




Consistent $\langle r^2 \rangle$ also for $G_M$

If the impact of projectile upon target is so energetic that target is fragmented, form factors are replaced by "Structure functions" to describe process ("Deep Inelastic Scattering")

$$d\sigma_{elastic} = L^e_{\mu\nu} L^{p\,\mu\nu}$$

$$\downarrow$$

$$d\sigma_{inelastic} = L^e_{\mu\nu} W^{\mu\nu}$$ it's no longer a proton

electron cannot dissociate

As projectile energy grows, cross section evolves from messy multibody (proton) to pointlike (quark) formula ⇒ "Scaling".

Feynman Rules for QCD.

Similar to QED (quarks are fermions like electrons)
{i.e., use $u, \bar{u}, v, \bar{v}$}

except:

quarks also carry wavefunction for color, so each $u$ or $v$ is multiplied by a vector

$$\begin{pmatrix}1\\0\\0\end{pmatrix} \text{-or-} \begin{pmatrix}0\\1\\0\end{pmatrix} \text{-or-} \begin{pmatrix}0\\0\\1\end{pmatrix}$$

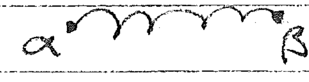
red blue green

and each $\bar{u}$ or $\bar{v}$ is multiplied by the rows $(1\ 0\ 0)$
$(0\ 1\ 0)$
$(0\ 0\ 1)$

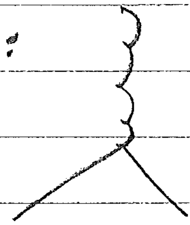
Gluons carry polarization $\varepsilon$ and color $a^{\alpha}$ ← incoming
$a^{\beta *}$ ← outgoing

Propagators: $q$ or $\bar{q}$: $\dfrac{i(\not{q}+m)}{q^2-m^2}$ recall gluons change quarks' colors

gluon: $\dfrac{-ig_{\mu\nu}\delta^{\alpha\beta}}{q^2}$



$g = \sqrt{4\pi\alpha_s}$

Vertices:  $-i\frac{g}{2}\lambda^\alpha\gamma^\mu$

$\lambda^\alpha$ are the Gell-Mann matrices, $3\times3$ analogs to the $2\times2$ Pauli matrices (See Griffiths Eq. 8.34)

$$\lambda^1 = \begin{pmatrix} & 1 & \\ 1 & & \\ & & \end{pmatrix} \qquad \lambda^7 = \begin{pmatrix} & & \\ & & -i \\ & +i & \end{pmatrix}$$

$$\lambda^2 = \begin{pmatrix} & -i & \\ +i & & \\ & & \end{pmatrix} \qquad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

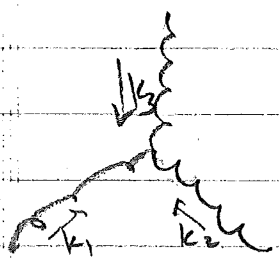
$$\lambda^3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & \end{pmatrix}$$

A representation of $SU(3)$

$$\lambda^4 = \begin{pmatrix} & & 1 \\ & & \\ 1 & & \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} & & -i \\ & & \\ +i & & \end{pmatrix}$$

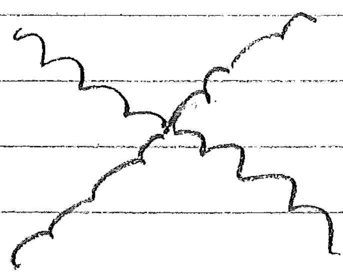
$$\lambda^6 = \begin{pmatrix} & & \\ & & 1 \\ & 1 & \end{pmatrix}$$




$$-gf^{\alpha\beta\gamma}\left[g_{\mu\nu}(k_1-k_2)_\lambda + g_{\nu\lambda}(k_2-k_3)_\mu + g_{\lambda\nu}(k_3-k_1)_\nu\right]$$

$f^{\alpha\beta\gamma}$ are the structure constants of SU(3). They encode the commutation relations of the matrices (recall these lead to antiscreening)

$$[\lambda^\alpha, \lambda^\beta] = 2if^{\alpha\beta\gamma}\lambda_\gamma$$



$$-g^2\left[f^{\alpha\beta\eta}f^{\gamma\delta\eta}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) + f^{\alpha\delta\eta}f^{\beta\gamma\eta}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho}) + f^{\alpha\gamma\eta}f^{\delta\beta\eta}(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\lambda\rho})\right]$$

– We reluctantly skip Ch 9 (Weak Int) and Ch 11 ($\nu$s) this semester. But for completeness:

Feynman rules for weak interactions:
Same as for QED except

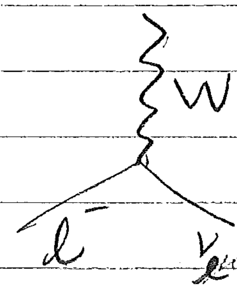
(1) massive $(M)$ propagator Z or W:

$$\frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2}\right)}{q^2 - M^2}$$

(2) Charged leptonic vertex

Charged → W

leptonic → $e, \mu, \tau$




$$\frac{-i g_W}{2\sqrt{2}} \gamma^\mu \underbrace{(1 - \gamma^5)}_{\text{"V-A"}}$$

$\gamma^5$ ← parity violation

1 ← parity is not 100% violated

"V-A"
vector-axialvector interaction

$g_W = 0.653 = \sqrt{4\pi \alpha_W}$