Note groups can be infinite or finite

set of integers under addition — order infinite

2 elements $\{+1, -1\}$. This is realized as parity — order 2

Most transformations can be represented by matrices.
Ex: rotation. Each member of this group can be expressed as

$$R_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

The matrix is a <u>representation</u> of the group.

If the matrix is not block-diagonal, "everything mixes with everything else" — no parts of the system are unaffected by the transformation — the representation is "irreducible".

Otherwise: $\begin{pmatrix} \square & 0 \\ 0 & \square \end{pmatrix}$: reducible.

A single group can have representations of different dimensions.

Ex. Consider the spin raising operator S_+. Recall $S_+|s\,m_s\rangle = \hbar\sqrt{s(s+1) - m(m+1)}\,|s, m_s+1\rangle$

$S_+|1,1\rangle = 0$

$S_+|1,0\rangle = \hbar\sqrt{2}\,|1,1\rangle$

$S_+|1,-1\rangle = \hbar\sqrt{2}\,|1,0\rangle$

$S_+|\tfrac{3}{2}, +\tfrac{3}{2}\rangle = 0$

$S_+|\tfrac{3}{2}, +\tfrac{1}{2}\rangle = \hbar\sqrt{3}\,|\tfrac{3}{2}, \tfrac{3}{2}\rangle$

$S_+|\tfrac{3}{2}, -\tfrac{1}{2}\rangle = \hbar\cdot 2\,|\tfrac{3}{2}, \tfrac{1}{2}\rangle$

$S_+|\tfrac{3}{2}, -\tfrac{3}{2}\rangle = \hbar\sqrt{3}\,|\tfrac{3}{2}, -\tfrac{1}{2}\rangle$

$\downarrow$ $\downarrow$

$$\begin{array}{c|ccc} & |1\rangle & |0\rangle & |-1\rangle \\ \langle 1| & 0 & \hbar\sqrt{2} & 0 \\ \langle 0| & 0 & 0 & \hbar\sqrt{2} \\ \langle -1| & 0 & 0 & 0 \end{array} \qquad \begin{array}{c|cccc} & |3/2\rangle & |1/2\rangle & |-1/2\rangle & |-3/2\rangle \\ \langle 3/2| & 0 & \hbar\sqrt{3} & 0 & 0 \\ \langle 1/2| & 0 & 0 & \hbar 2 & 0 \\ \langle -1/2| & 0 & 0 & 0 & \hbar\sqrt{3} \\ \langle -3/2| & 0 & 0 & 0 & 0 \end{array}$$

Group theory is applied in physics to symmetries of
- geometry (crystals, historically first)
- permutation (isospin, spin, interchange of quark types in Eightfold Way diagrams . . .)

The direct product of 2 irreducible representations produces a reducible representation that can be decomposed into multiple irreducible ones:

Cartesian product!! set of all possible ordered pairs

→ Ex from spin ½: $2 \otimes 2 = 3 \oplus 1$

Combine 2 spin ½ quarks into a meson: Each is separately a 2-D object to accommodate $s = +\hbar/2$ and $s = -\hbar/2$

Possible combinations are

Ordered as: $|s_1 m_{s_1}\rangle |s_2 m_{s_2}\rangle$

$$\left.\begin{array}{l} |\tfrac{1}{2}\,\tfrac{1}{2}\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = |1\,1\rangle \\ \tfrac{1}{\sqrt{2}}\left\{|\tfrac{1}{2}\,\tfrac{1}{2}\rangle|\tfrac{1}{2},\tfrac{-1}{2}\rangle + |\tfrac{1}{2}\,\tfrac{-1}{2}\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle\right\} \\ |\tfrac{1}{2},\tfrac{-1}{2}\rangle|\tfrac{1}{2},\tfrac{-1}{2}\rangle = |1,-1\rangle \end{array}\right\} \text{3 symmetric compound wavefns}$$

$$\tfrac{1}{\sqrt{2}}\left\{|\tfrac{1}{2}\,\tfrac{1}{2}\rangle|\tfrac{1}{2}\,\tfrac{-1}{2}\rangle - |\tfrac{1}{2}\,\tfrac{-1}{2}\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle\right\} \quad \text{1 antisymmetric compound wavefn}$$

The result is a 3-D and a 1-D rep.

Ex from QCD, 3 colors $3 \otimes 3 = 8 \oplus 1$ ← not realized in nature

These are the eight colored gluons

Ex from Eightfold Way, 3 light quarks (u, d, s)

$3 \otimes 3 = 8 \oplus 1$ light meson nonet

$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$ ← antisym forbidden by Fermi statistics

Re: Gell-Mann's prediction of the Ω

When we combine 2 irreducible rep's into a larger reducible rep., ex -

$$|j_1 m_1\rangle |j_2 m_2\rangle = \sum{}' C \, |j m\rangle$$

The coefficients of the transformation are the Clebsch-Gordon coefficients. Individual coefficients predict probability amplitudes → branching ratios

I Isospin

Consider a space in which similar mass particles are grouped into multiplets

$$\begin{pmatrix} p \\ n \end{pmatrix} \quad \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \quad \text{etc.} \qquad \text{"Isospin space"}$$

A rotation in this space transforms particles from one type to the other.

Describe this with math analogous to that for spin, so define an Isospin-z component that distinguishes the members of the multiplet.

The associated group for spin or isospin is $SU(2)$. (det! unitary 2-dimen)

Since $\begin{pmatrix} p \\ n \end{pmatrix}$ has 2 members, it has $I = \frac{1}{2}$ (no $\hbar$)

multiplicity = $2I+1$, solve for I.

Choose $p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (order by electric charge)

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$I_z = +\tfrac{1}{2} \qquad\qquad I_z = -\tfrac{1}{2}$$

(I_x and I_y have no meaning)

Similarly the pion multiplet has $I = 1$, $I_3 = \{+1, 0, -1\}$

Heisenberg proposed that strong interactions conserve I and I_3. (Data show that he was right)

Use this and Clebsch Gordan coef. to predict cross sections for various strong scatters:

Compare $pp \to d\,\pi^+$

$pn \to d\,\pi^0$

$nn \to d\,\pi^-$

Write isospin vectors of the composite states on the left side

$$\left.\begin{aligned} |1\,1\rangle &= pp \\ |1\,{-1}\rangle &= nn \\ |1,0\rangle &= \tfrac{1}{\sqrt{2}}\{pn + np\} \end{aligned}\right\}$$ all are symmetric wrt exchange of particles 1, 2

to include it in the symmetric multiplet

$|0,0\rangle = \frac{1}{\sqrt{2}}\{pn - np\}$ antisymmetric

only 1 member of the antisym multiplet.

Now on the right side:

deuteron $d = |0\,0\rangle$, so isospin states on RHS are determined by the pions

$d\,\pi^+ = |1\,1\rangle$

$d\,\pi^0 = |1\,0\rangle$

$d\,\pi^- = |1\,{-1}\rangle$

Notice $pn = \frac{1}{\sqrt{2}}\{|1\,0\rangle + |0\,1\rangle\}$

If isospin is conserved (Heisenberg's conjecture) then only the $\frac{1}{\sqrt{2}}|1\,0\rangle$ amplitude contributes to production of the $|1\,0\rangle$ $(d\pi^0)$ final state

See Wikipedia: Grand Unified Theory

So we expect the relative amplitudes for these 3 otherwise similar processes to be

$$1 : \frac{1}{\sqrt{2}} : 1$$

Cross section ~ $|\text{Amplitude}|^2$ so what is actually measured is

$$1^2 : \left(\frac{1}{\sqrt{2}}\right)^2 : 1^2 \rightarrow$$

$$1 : \frac{1}{2} : 1$$ in the data.

Note: The goal of grand unification is to identify every force with a group, eg.

Strong: $SU(3)$ ⟸ non-Abelian, leads to gluon self coupling
weak: $SU(2)$
EM: $U(1)$

Then find a larger group that has these as subgroups. For example $SU(5)$ and $SO(10)$ include these

Both of these decompose to irreducible reps that include a 5-D one with 3 quark colors + lepton doublet $\begin{pmatrix} l^+ \\ \nu_e \end{pmatrix}$, for each generation.

This is why transformations between q and l are a typical prediction of GUT.

This is a Lie group

Consider the elements of a group, including I.
Assume that in the neighborhood of the identity I,
the elements can be described by a function of
parameters α_i $(i=1,\ldots N)$ that limits to I when $\alpha=0$.
Expect the same of a representation.
Taylor expand the representation D

$$D(d\alpha) = 1 - i\,d\alpha\, X_\alpha + \cdots$$

assures unitarity

So $X = i \left.\frac{\partial D}{\partial \alpha}\right|_{\alpha=0}$

The X are the generators of the group

We recognize this as the series rep. of the exponential function

$$D(\alpha) = e^{-iX\alpha}$$

Special case, when $\left\{ \begin{array}{l} D = \text{translation matrices,} \\ X = \text{momentum } P \end{array} \right.$

or " $\left\{ \begin{array}{l} D = \text{rotation} \\ X = \text{ang. mom. } J \end{array} \right.$

It turns out that the generator [of groups of this type (Lie)] of a group of transformations is the QM operator.

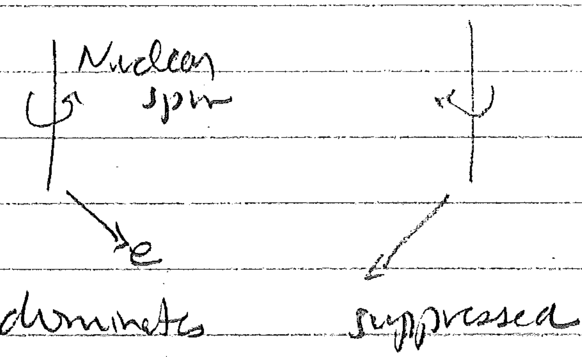
Nobel Prize Women in Science
Sharon Bertsch McGrayne

I. Discrete symmetries
involving discontinuous transformations, transformation associated with finite groups

Parity - reflection through the origin

Violated by the weak force, conserved by strong + EM
CS Wu experiment, 1956:

$^{60}Co \rightarrow {}^{60}Ni + e + \bar{\nu}_e$



Define helicity = direction of particle's spin z component relative to " momentum vector

same direction: right handed
opp left

Note this is changed by a Lorentz boost if particle has $v < c$ (i.e. $m \neq 0$)
prior to the discovery of ν mass it was believed that all ν are left handed
(antiparticle parity is opposite for fermions
" " same " bosons)

Note the parity operator has 2 eigenvalues, ±1

$P^2\psi = 1\psi$, so $P = \sqrt{1}$

Quantities can be categorized wrt their parity

scalars - 1 comp obj that is parity even
pseudoscalars - 1 component object that is parity odd
vectors - 3 comp " " " " odd
pseudovectors - 3 component object that is parity even
tensors

$P(\text{quark}) = +1$
$(\bar{q}) \quad -1$
$(q\bar{q}) = ((+1)(-1)\cdot(-1)^{\ell} \leftarrow Y_{\ell m}$

- Charge conjugation

$C|p\rangle = |\bar{p}\rangle$ applies to neutral as well as charged particles

$C|\gamma\rangle = -|\gamma\rangle$ because EM field changes sign under C if we take charge to be scalar not pseudoscalar

$C(q\bar{q}) = (-1)^{\ell+s}$

Violated by weak, conserved by em + strong

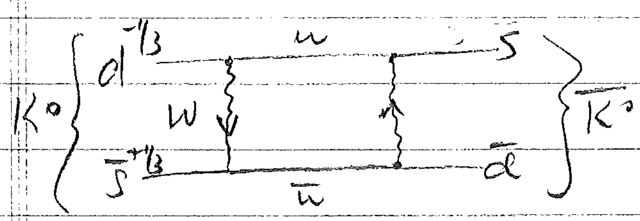
- CP

  Also violated by weak interactions

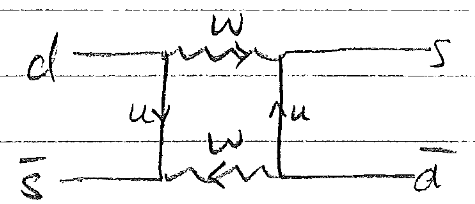
- CPT : required by Lorentz invariance + locality. A few tiny hints of violation

I. Kaon mixing

This is possible:



and



Notice

$P|K^0\rangle = -|K^0\rangle$

$P|\overline{K^0}\rangle = -|\overline{K^0}\rangle$

$C|K^0\rangle = |\overline{K^0}\rangle$

$C|\overline{K^0}\rangle = K^0$

So $CP|K^0\rangle = -|\overline{K^0}\rangle$

$CP|\overline{K^0}\rangle = -|K^0\rangle$

So neither K^0 nor $\overline{K^0}$ is an eigenstate of the CP operator, however

$$|K_1\rangle \equiv \frac{1}{\sqrt{2}}\{|K^0\rangle - |\bar{K}^0\rangle\}$$ and

$$|K_2\rangle \equiv \frac{1}{\sqrt{2}}\{|K^0\rangle + |\bar{K}^0\rangle\}$$

are eigenstates

Notice

$CP|K_1\rangle = +|K_1\rangle$ "CP even"
$CP|K_2\rangle = -|K_2\rangle$ "CP odd"

This is an example of alternate basis representation for a 2-level QM state
K's decay to pions! { K_1-K_2 versus $K_0\bar{K}_0$. Analogous to the tunneling NH_3 molecule }

$K \to n\pi$ what is n?

If 2 pions, They must be $\pi^0\pi^0$ or $\pi^+\pi^-$ because $K_{1,2}$ are neutral.

Recall $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{d} + d\bar{u})$

$\pi^+ = u\bar{d}$

$\pi^- = \bar{u}d$

So $C(\pi\pi) = +(\pi\pi)$
Also $P(\pi\pi) = P(\pi)P(\pi) = (-1)(-1) = +1$
So $CP(\pi\pi) = +(\pi\pi)$

Jan 12, 1987 $57 \text{ ft} \times \frac{12 \text{ in}}{\text{ft}} \times \frac{.0254 \text{ m}}{\text{in}}$

Similarly $CP(\pi\pi\pi) = -(\pi\pi\pi)$

Expect, if CP is conserved, that

$K_1 \to 2\pi$

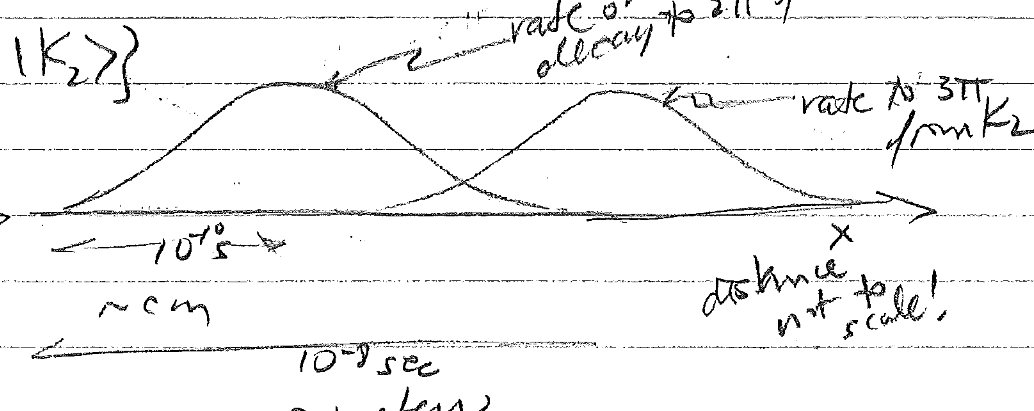
$K_2 \to 3\pi$

2π decay has more phase space, happens faster.

Notice

$|K^0\rangle = \frac{1}{\sqrt{2}}\{|K_1\rangle + |K_2\rangle\}$

So predict: K^0 beam at $v \approx c$



Cronin + Fitch (1964) looked at distance 17 m and found 45 2π events out of 22700 total (a 0.2% effect)

$\Rightarrow$ CP violation, Nobel 1980

So the true CP-even eigenstate is mostly K_1 but a little K_2

short lived $\rightarrow$ $K_S = \frac{1}{\sqrt{1+|\varepsilon|^2}}\{|K_1\rangle + \varepsilon|K_2\rangle\}$ $\qquad \varepsilon = 2.2 \times 10^{-3}$

And the true CP-odd eigenstate is:

long lived $\rightarrow$ $K_L \equiv \frac{1}{\sqrt{1+|\varepsilon|^2}}\{\varepsilon|K_1\rangle + |K_2\rangle\}$ $\qquad$ oddly small number

3 Interesting related topics:

(1) "Philosophical question: what defines a particle"?
(i) How it's produced? → $K^0, \bar{K}^0$ (strong)
(ii) The operator for which it is an eigenstate? → K_1, K_2 (CP
(iii) What is detected? → K_S, K_L

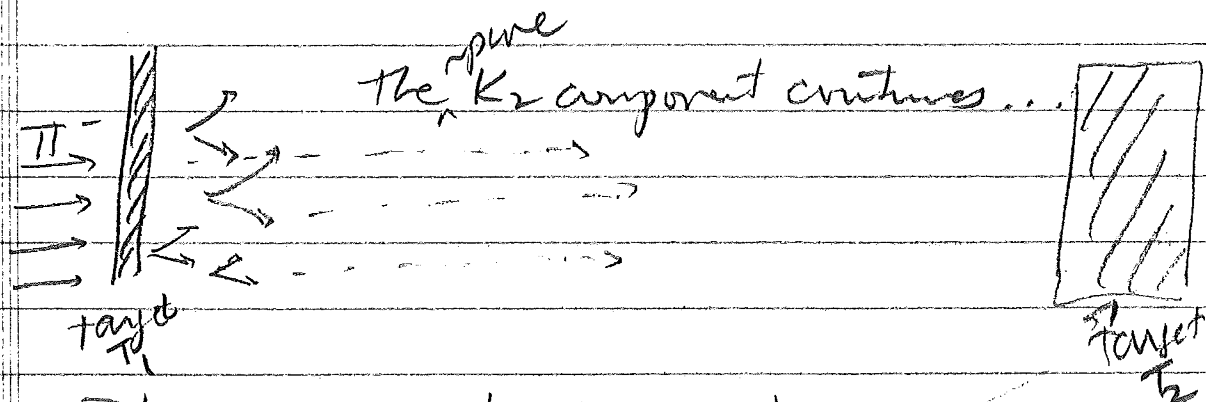
(2) Regeneration

Collide p with a target, extract a π^- beam.
" π^- with a thin target, produce a K^0 beam

$$\pi^- p \to K^0 \Lambda$$

Note $|K^0\rangle \propto \{|K_1\rangle + |K_2\rangle\}$

Let K_1's decay, $K_1 \to \pi\pi$ observed directly after the target



Place a second target downstream
Note the $K_2 \cong \frac{1}{\sqrt{2}}\{K^0 + \bar{K}^0\}$

The K^0 and $\bar{K}^0$ components interact differently with the second target, because

K^0 has strangeness +1
$\bar{K}^0$ " " −1
} and the strong force conserves strangeness.

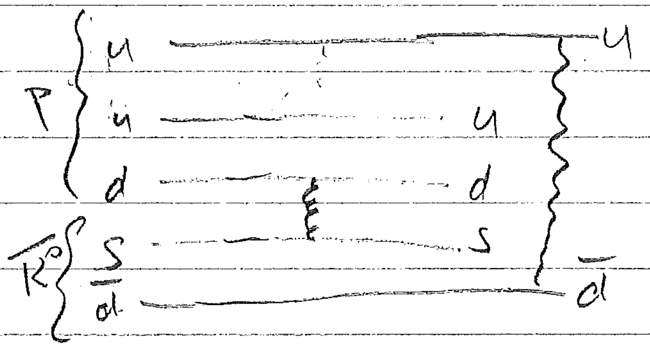
$\binom{d}{\bar{s}}$ $\binom{u}{\bar{s}}$

So: $K^\circ p \rightarrow K^+ n$ is the only possible interaction for K°,

But $\bar{K}^\circ$ has more channels available

$\binom{s}{\bar{d}}$ $\left(\begin{smallmatrix} s \\ u \\ d \end{smallmatrix}\right)$

$\bar{K}^\circ p \rightarrow \Lambda \pi^+$



$\bar{K}^\circ p \rightarrow \Sigma^\circ \pi^+$

$\bar{K}^\circ n \rightarrow \Lambda^\circ \pi^\circ$

$\bar{K}^\circ n \rightarrow \Sigma^\circ \pi^\circ$

$\bar{K}^\circ p \rightarrow K^+ n$

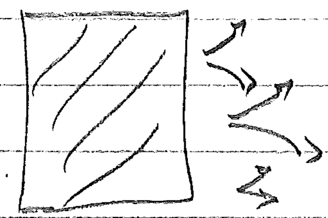
Note $K_\circ p \rightarrow \bar{\Lambda} \pi^+$ does not work to first order:

u	$\bar{u}$
u	$\bar{d}$
d	$\bar{s}$
$\bar{s}$	u
d	$\bar{d}$

So by extending the length of target T_2, can make $\bar{K}^\circ$ component of the K_2's $\longrightarrow 0$ leaving K° with intensity $1/4$ of initial

But $K^\circ = \frac{1}{\sqrt{2}}\{K_1 + K_2\}$

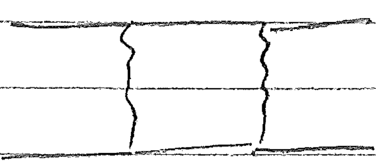
So immediately beyond the target T_2, there is a renewed $K_1 \to \pi\pi$ decay signal,



"K_1 regeneration" often called K_S^0 regeneration:
How many K_1's?
Suppose fraction f of the K^0's survive T_2
and " $\bar{f}$ . . . $\bar{K}^0$'s . . . .
where $\bar{f} < f < 1$
Then the $|\text{regenerated}\rangle = \frac{1}{\sqrt{2}}\left\{ f|K^0\rangle + \bar{f}|\bar{K}^0\rangle \right\}$

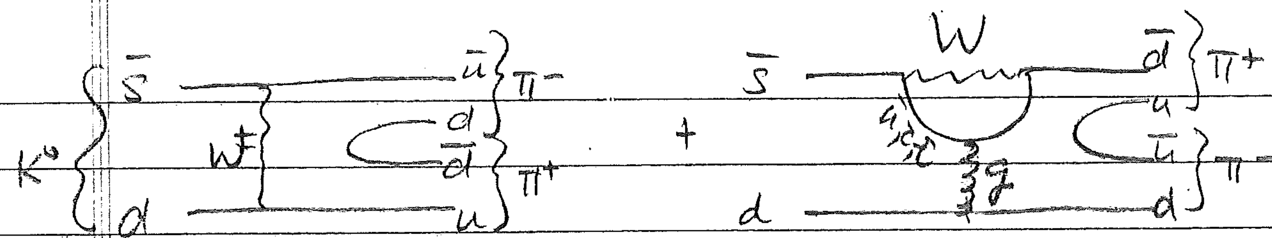
$$= \frac{1}{2}(f-\bar{f})|K_1\rangle + \frac{1}{2}(f+\bar{f})|K_2\rangle$$

(3) Kinds of CP violation



(box diagram): "Indirect CP violation" due to mixing

But also: CP violation arises from interference between tree-level and Penguin diagrams leading to the same final state $\leftarrow \sigma = |A_1 + A_2|^2$



"Direct CP Violation" characterized by ε'

There is separate tabulation of

$$|\eta_{+-}| = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \quad \text{and} \quad |\eta_{00}| = \frac{A(\quad \pi^0\pi^0)}{A(\quad \pi^0\pi^0)}$$

Ch5 - Bound states

(We are not covering the basic QM in this chapter)

• Positronium e^+e^- bound, decays to $\gamma\gamma$ in 10^{-10}s
PET

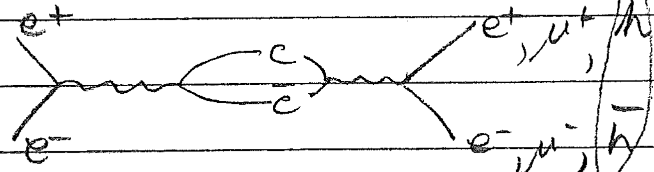
measurement of its levels provides precision test of QED as there's no uncertainty due to proton structure as in hydrogen.

• Quarkonium

$q\bar{q}$ (same flavor) bound state

The first $c\bar{c}$ resonance found was J/ψ (1974)

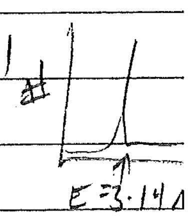
@ SLAC



@ BNL $pp \to J/\psi \to e^+e^- X$

Be target

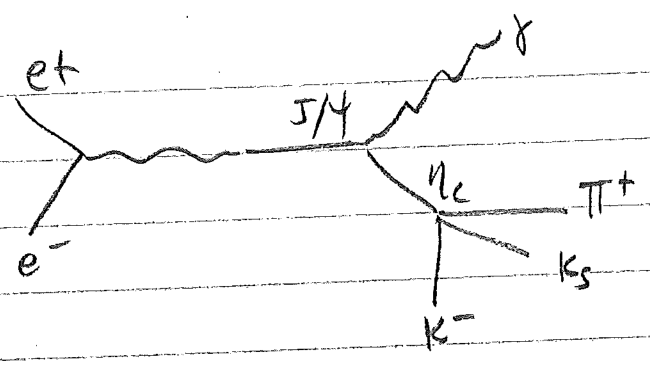
Narrow peak due to OZI



Because J/ψ is produced by a γ (ang. mom = 1), and ang. mom is conserved by all interactions, the J/ψ has ang. mom = 1. So it's <u>not</u> the ground state of its system.

The ground state is called η_c. Why wasn't it discovered first?

Reported Oct 1980



Mixing c → s 97%

$$\begin{pmatrix} V & V & V \\ V & V_{cs} & V \\ V & V & V \end{pmatrix}$$

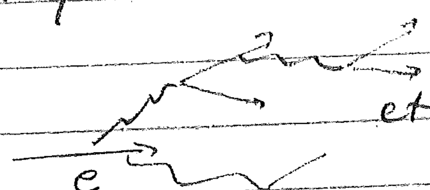
Look for excess of γ's with particular energy E
(where $E = m_{J/\psi} - m_\eta$)

Very low energy γ: 3096 MeV
− 2980 MeV
114 MeV

(PEP storage ring e^+e^- com energy ~7.4 GeV)

was discovered by NaI calorimeter experiment
"The Crystal Ball" @ SLAC

Optimized for measurement of showering particles



Material's characteristic probability to induce a pair, combined with its char. multiple scattering angle, is its <u>radiation length X_0</u>

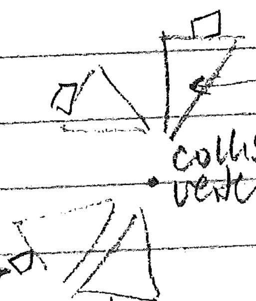
$$\frac{1}{X_0} = \frac{4\alpha N_A Z(Z+1) r_e^2 \log(183 Z^{-1/3})}{A}$$

at. # — Avogadro's — 2.8×10^{-13} cm — at. wt.

The shower's induced by interactions with calorimeter e's

A few words about (EM) calorimetry

(1)



collision vertex

Towers in "projective" geometry point back to the vertex.

Tower depth sufficient to fully absorb energy (no particles "punch through the back")

(2) Crystals wrapped light-tight, light is fully channeled to PMT's

that were detected in both spark chambers, both direction and origin along the beam line could be determined. Those that failed to be detected in both spark chambers were only "tagged," i.e. identified as being charged.

The heart of the detector, of course, was the spherical shell, 16 radiation lengths thick, made of sodium iodide. This thickness is sufficient to contain essentially the entire longitudinal development of electromagnetic showers in our energy range. As shown in Figure 2, the shell is actually a dense packing of truncated triangular pyramids of NaI(Tl). These are optically

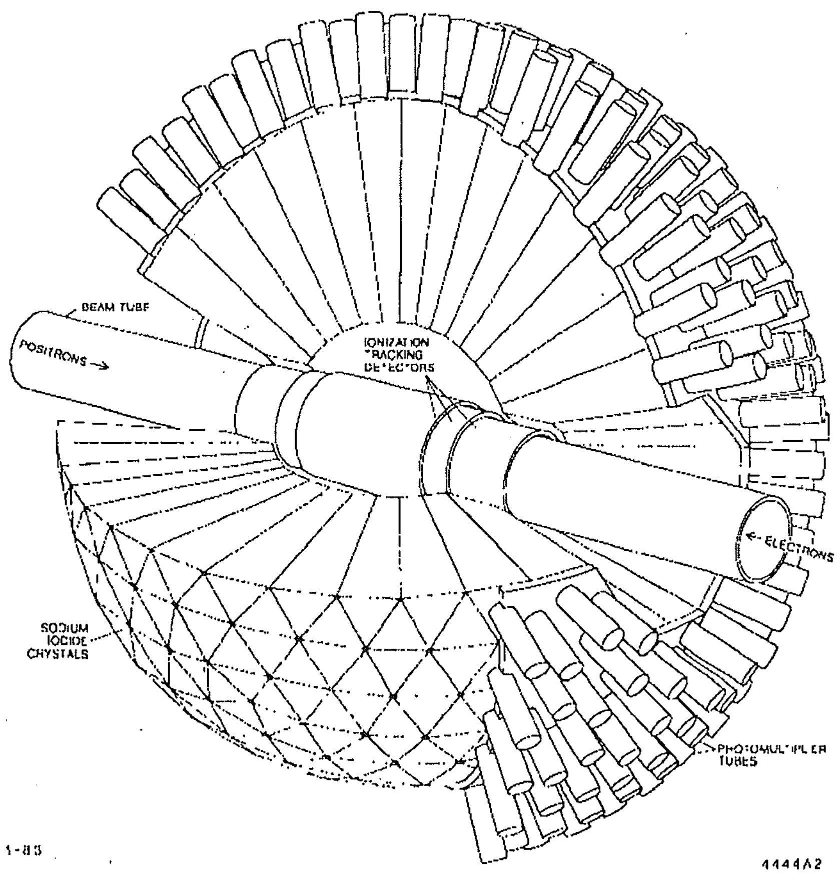


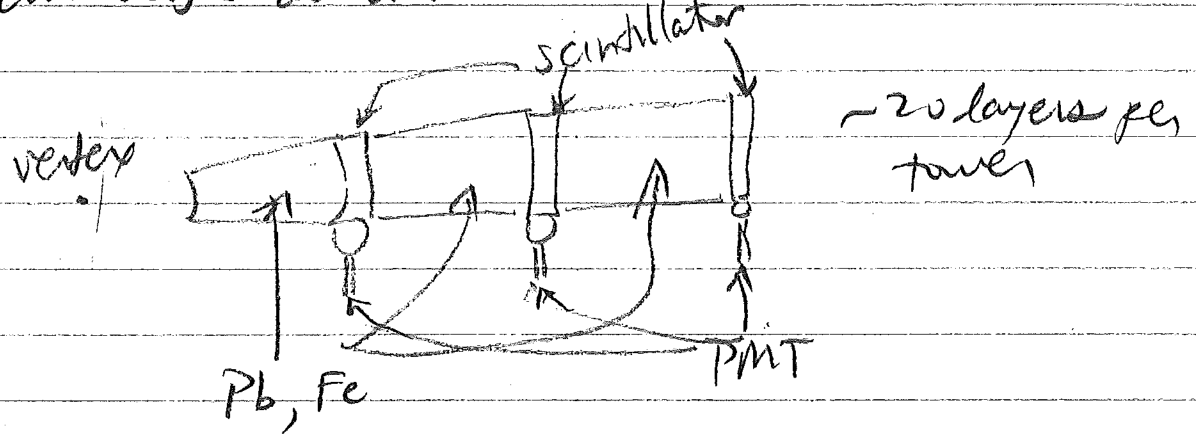
Figure 2 The two principal elements of the Crystal Ball detector, the charged-particle tracking chambers in the 25-cm diameter cavity of the shell, and the NaI(Tl) shell itself. The middle chamber is a continuously sensitive wire proportional chamber and the other two are magnetostrictive spark chambers. The shell itself is segmented into optically separated triangular pyramids in a solidly packed geometry based on an icosahedron. Each pyramid is viewed from the outside by a single photomultiplier. [From *Quarkonium*, by E. D. Bloom and G. J. Feldman. Copyright © 1982 by Scientific American, Inc. All rights reserved.]

(3) Energy of primary particle is proportional to # charged particles in shower and pathlength

(4) Design choices for the shower medium.

(i) This uses shower medium: transparency
The Crystal Ball used NaI → scintillation light
If medium were lead glass → Cherenkov light
This is called "homogeneous detector"

(ii) Another option is an opaque radiator interleaved with light detectors



This images the full shower development: "a sampling calorimeter"

Note resolution $R = \frac{\Delta E}{E}$. Photoelectrons are detected from a Poisson dist for which $\sigma^2 = \text{Mean}$, so $R = \frac{\sigma}{\text{Mean}} = \frac{1}{\sqrt{E}}$.

- Understanding meson masses.
  Modeled as $M_{TOT} = m_{q_1} + m_{q_2} + \underbrace{\frac{A}{m_1 m_2}(\vec{S}_1 \cdot \vec{S}_2)}$

  hyperfine

→ To understand the $\vec{S}_1 \cdot \vec{S}_2$ term:
Recall (Jackson p. 186)
$U = \vec{m} \cdot \vec{B}$