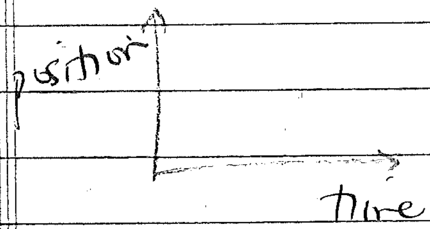
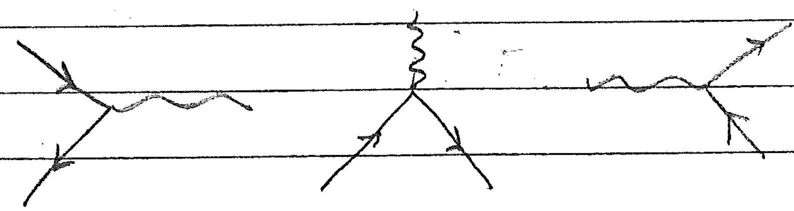
I. QED

Consider particle interactions graphed in the space



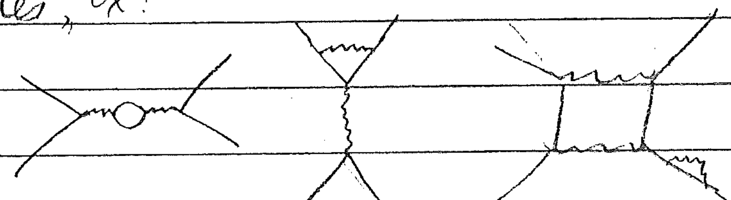
Let $\longrightarrow$: an electrically charged particle
$\longleftarrow$: " " " - antiparticle
$\sim$: a photon

Any process that involves 2 arrows (head to tail only) and 1 $\sim$ is allowed. Ex:



$q\bar{q} \to \gamma$ pair annihilation (or e^+e^-, etc.) | q emits photon (one side of an interaction) | $\gamma \to q\bar{q}$ pair creation

Call these "electromagnetic vertices."
Can build any complicated diagram that preserves these rules, ex:

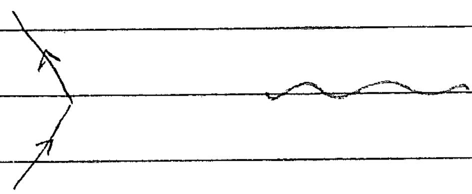


How do we know these are allowed? All are valid amplitudes (or diagrams) for (initial state)
solutions to a perturbative (series) expansion of the Schroedinger Eq.

Notes:

(0) No arrows on the γ lines as the γ is its own antiparticle so these represent transmission in either / both directions.

(1) A sole vertex is not a physical interaction because q or e cannot emit a γ and conserve $\vec{p}$



$\vec{p}_{com} = 0$ $\quad$ $\vec{p} \neq 0$, cannot transform to com for a massless particle (at $v = c$)

Conclude: internal lines of a diagram are not observable, the particles are "virtual": the particles are off mass shell
where the "shell" is the set of values allowed by

$$E^2 = m^2 + p^2$$

$$\downarrow$$

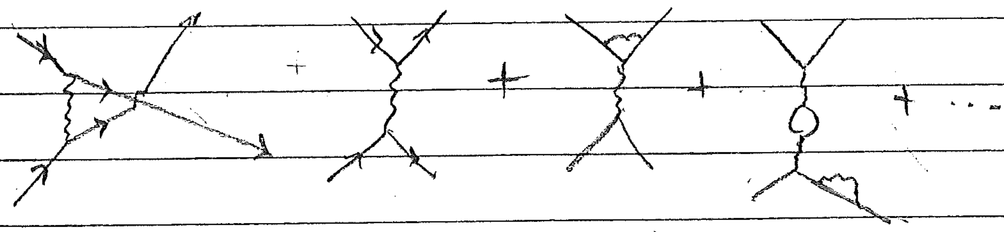
$$m^2 = E^2 - p^2 \neq 0 \text{ even though it is a photon}$$

Note paradox of photons emitted by stellar radiation. Since $\Delta E \Delta t \geq \hbar$, their $\Delta E = m$ must be very small to allow them to persist over $\Delta t \sim \frac{\hbar}{\Delta E}$ to reach our detectors.

Check: try drawing Compton scatter 

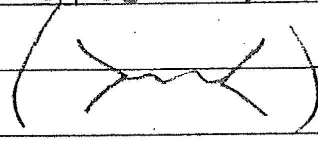
(2) Names

(i) $e^-e^- \to e^-e^-$ "Møller scattering" includes all of

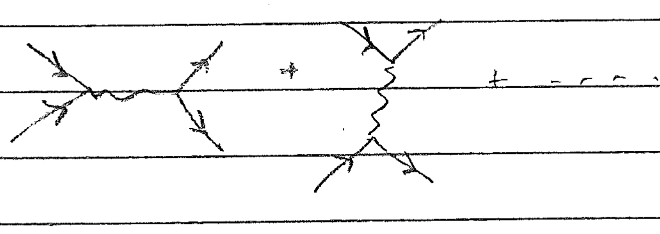


The series is infinite, however the contribution of each term to the total cross section $\propto \frac{1}{\#\text{vertices}}$ so it limits.

weight factor: $\alpha = \frac{1}{137}$ per vertex

Notice it is a true scatter — annihilation diagrams () are not possible

(ii) $e^+e^- \to e^+e^-$ "Bhabha scattering"



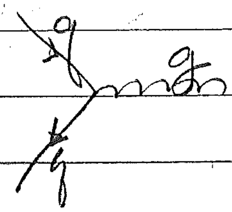
includes annihilation + scatter

How is it useful: The rate can be calculated to $\sim 10^{-4}$ so e^+e^- colliders can predict the rate + compare to data. The prediction requires an overall scale factor for #collisions (= "luminosity") $cm^{-2}s$ So the comparison to data determines this factor for the collider.
→ "luminosity monitoring"

II QCD

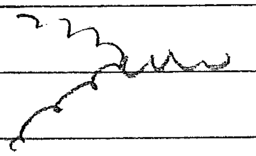
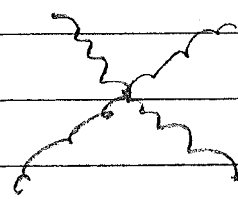
Modeled on QED, but more allowed vertices

Type #1

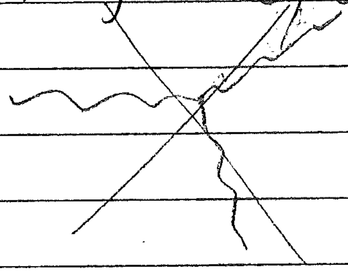


Note: Only q, not leptons participate

But also gluon self-coupling

 and , are allowed.

Note analogous self coupling of photon is not allowed



Why?

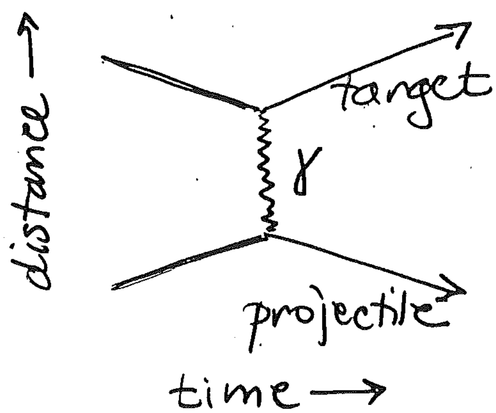
Gluon self coupling is added to the theory to account for the experimental observation that the strong charge (α_s) varies with the scale on which it is probed.

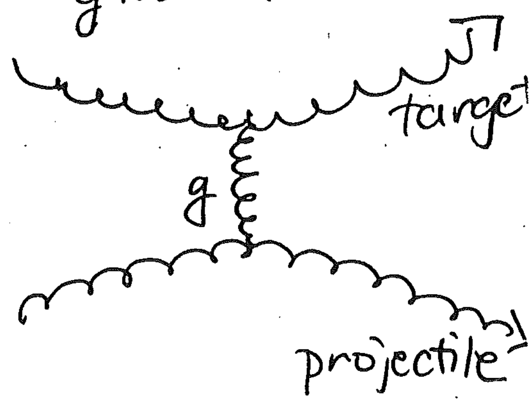
"The running of the strong coupling"!

How to visualize this:

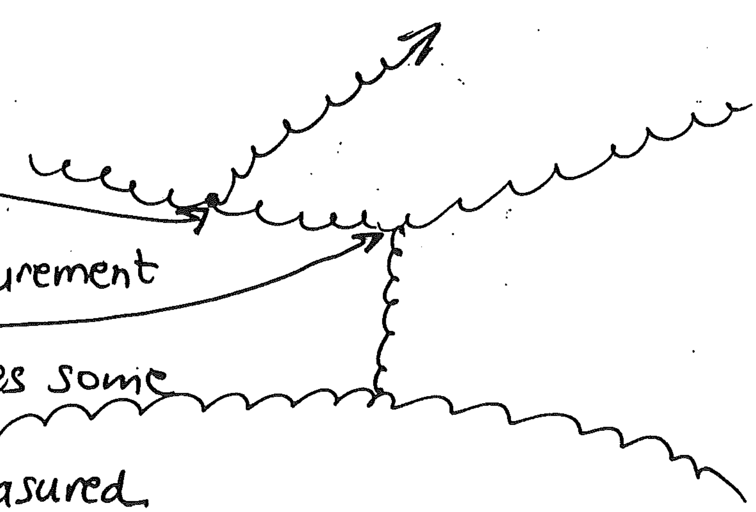
Recall (strength of) charge $\propto$ probability that the particle under test will emit the kind of boson that carries the force.

	EM force	Strong force
…pling …stant:	$\alpha \equiv \frac{e^2}{\hbar c}$	α_s
…roportional to:	electric charge "e"	color charge "g"
…ire a projectile …t a target	electron electron	gluon gluon
…this can happen:	The target emits photons:	The target emits gluons:



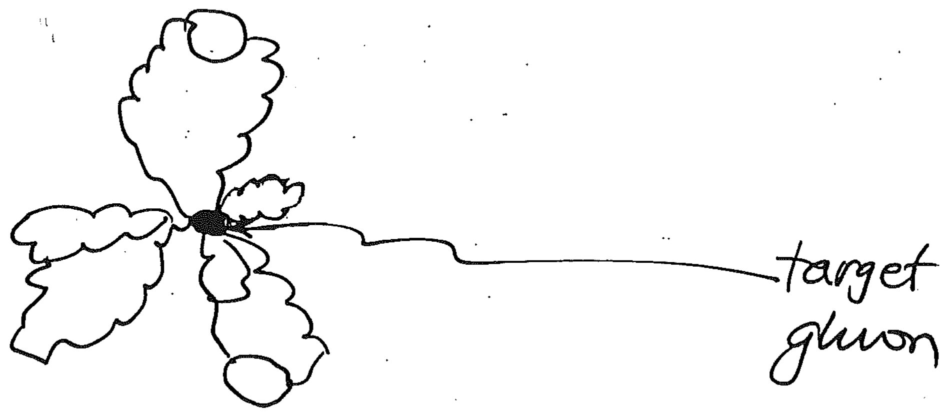


But also this can happen:

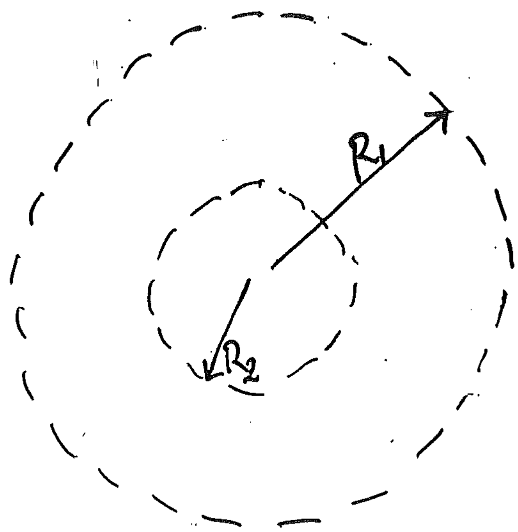


Before the measurement occurs here, the target radiates some of its color away here, so the measured strength appears weaker.

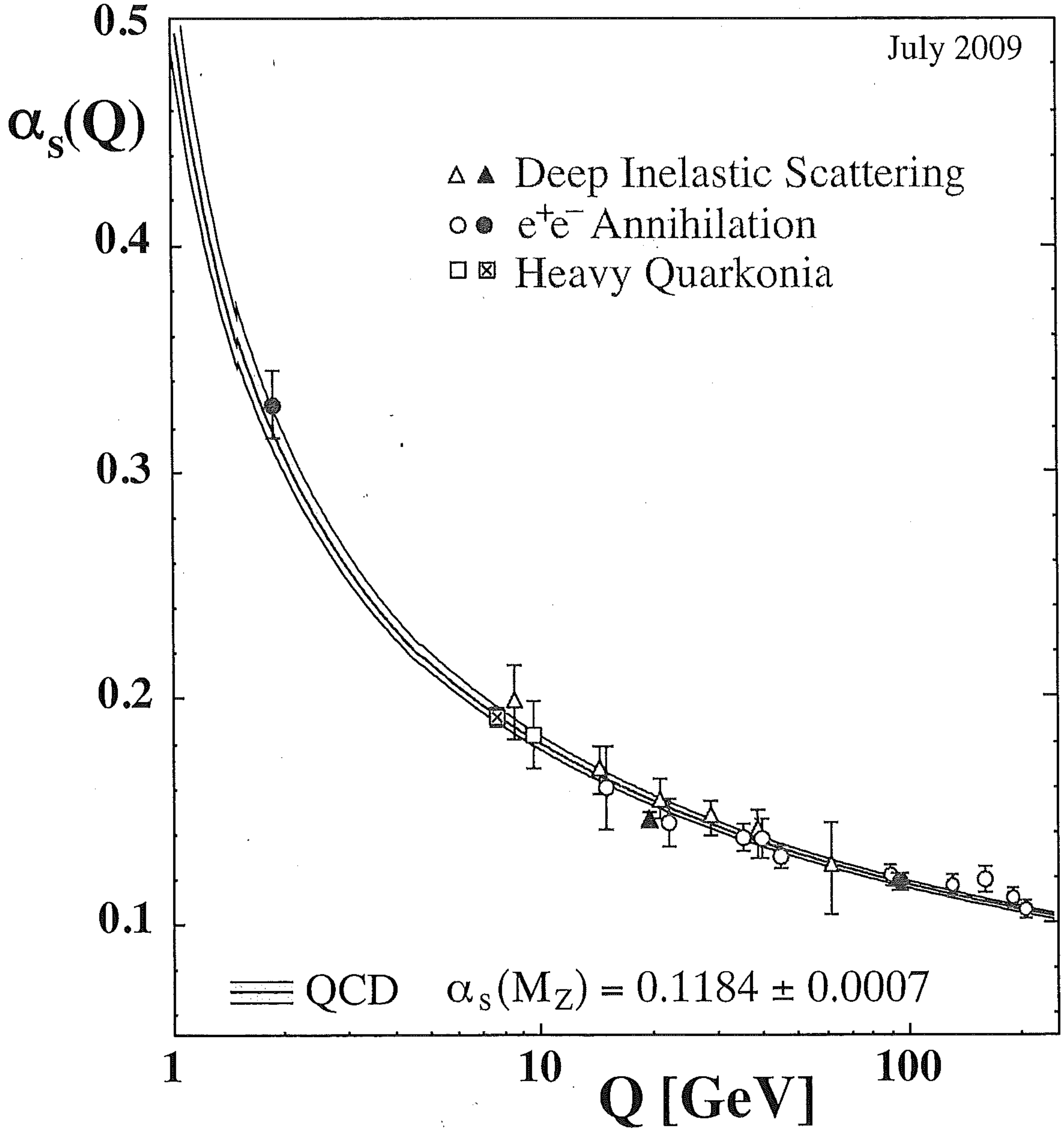
* The radiated gluons always eventually connect back to the target (* there are no free quarks/gluons), but the loops they make can be large:



so the closer the projectile comes to the target, the more likely it is to "miss some of the color."



A projectile that recoils @ radius R_1 will sense all of the color; a projectile that gets closer (to within R_2) will miss some color.

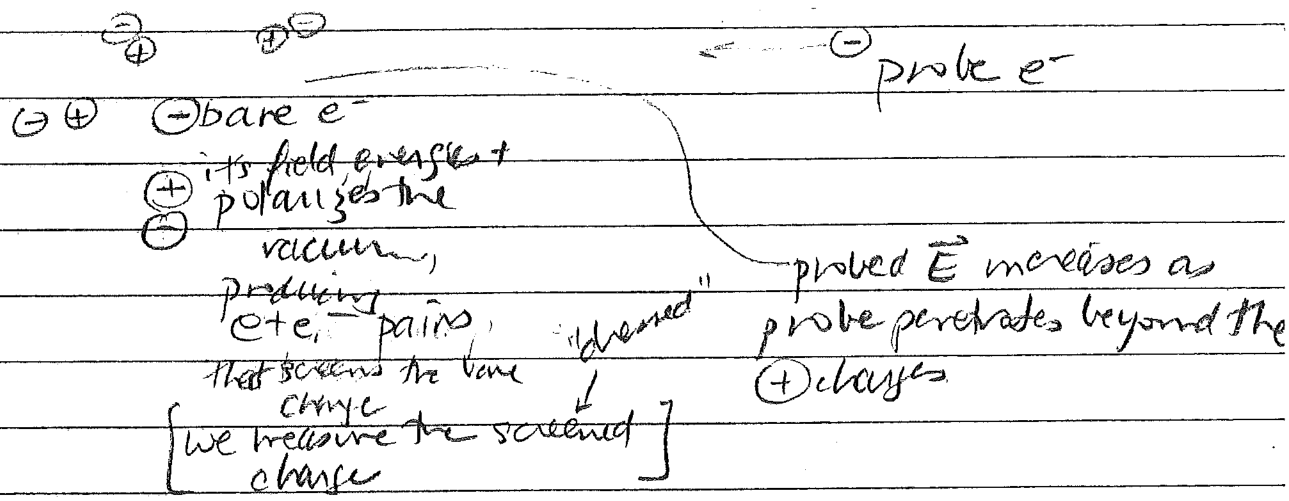


0.5
0.4
0.3
0.2
0.1
$\alpha_s(Q)$
July 2009
Deep Inelastic Scattering
e^+e^- Annihilation
Heavy Quarkonia
QCD $\alpha_s(M_Z) = 0.1184 \pm 0.0007$
1
10
100
Q [GeV]

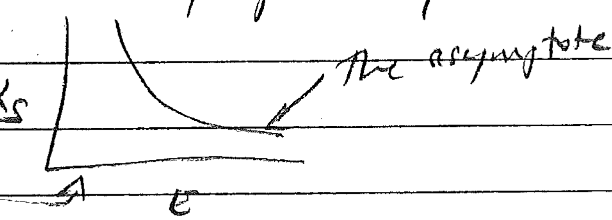
Vocabulary

(i) The effect that the closer the probe approaches, the weaker the charge is, is called <u>anti-screening</u>.

Compare electrostatics:

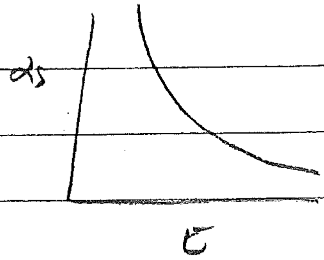


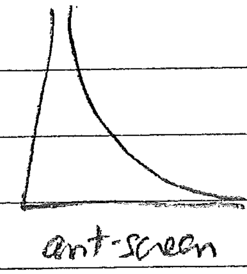
(ii) Regime of minimum α_s, very weak inter-quark field, (ultra-high energy of the probe): "asymptotic freedom"

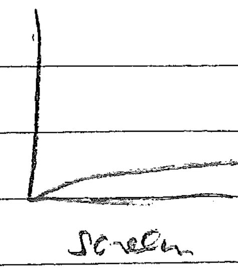


(iii) Regime of max α_s, strong q-$\bar{q}$ field (low energy probe): "infrared slavery" → may produce <u>confinement</u>.

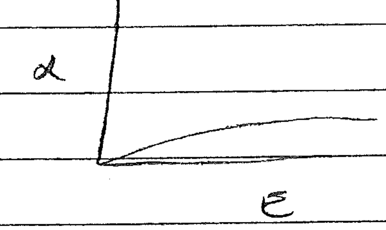
Quark color charge is anti-screened by the gluon loops, but also screened by virtual $q\bar{q}$ pairs analogously to electrostatics. The 2 effects compete, and the antiscreening dominates.

So α_s


is actually


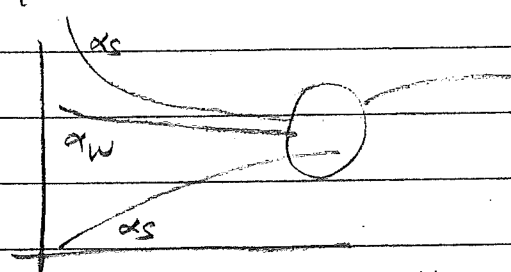
+


Note α also runs, but less dramatically:



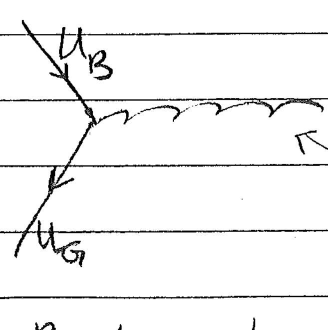
0.8% change as probe energy changes
from $2\ GeV^2 - 6\ GeV^2$
$1.4\ GeV - 2.5\ GeV$
α_s changes by 8% over same range

In fact the weak coupling also runs:

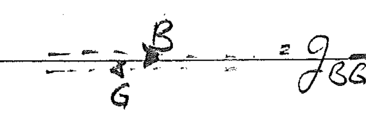


? do they meet?
Near $E = 10^{15}$ GeV they come very close but do not meet.
However one hypothesizes additional SUSY particles available to contribute to screening, then they do.

The fact that gluons carry color means the quark color can be changed by the interaction



redraw gluon as



$= g_{BG}$

The 8 gluon types are listed in Griffiths Eq. 8.29

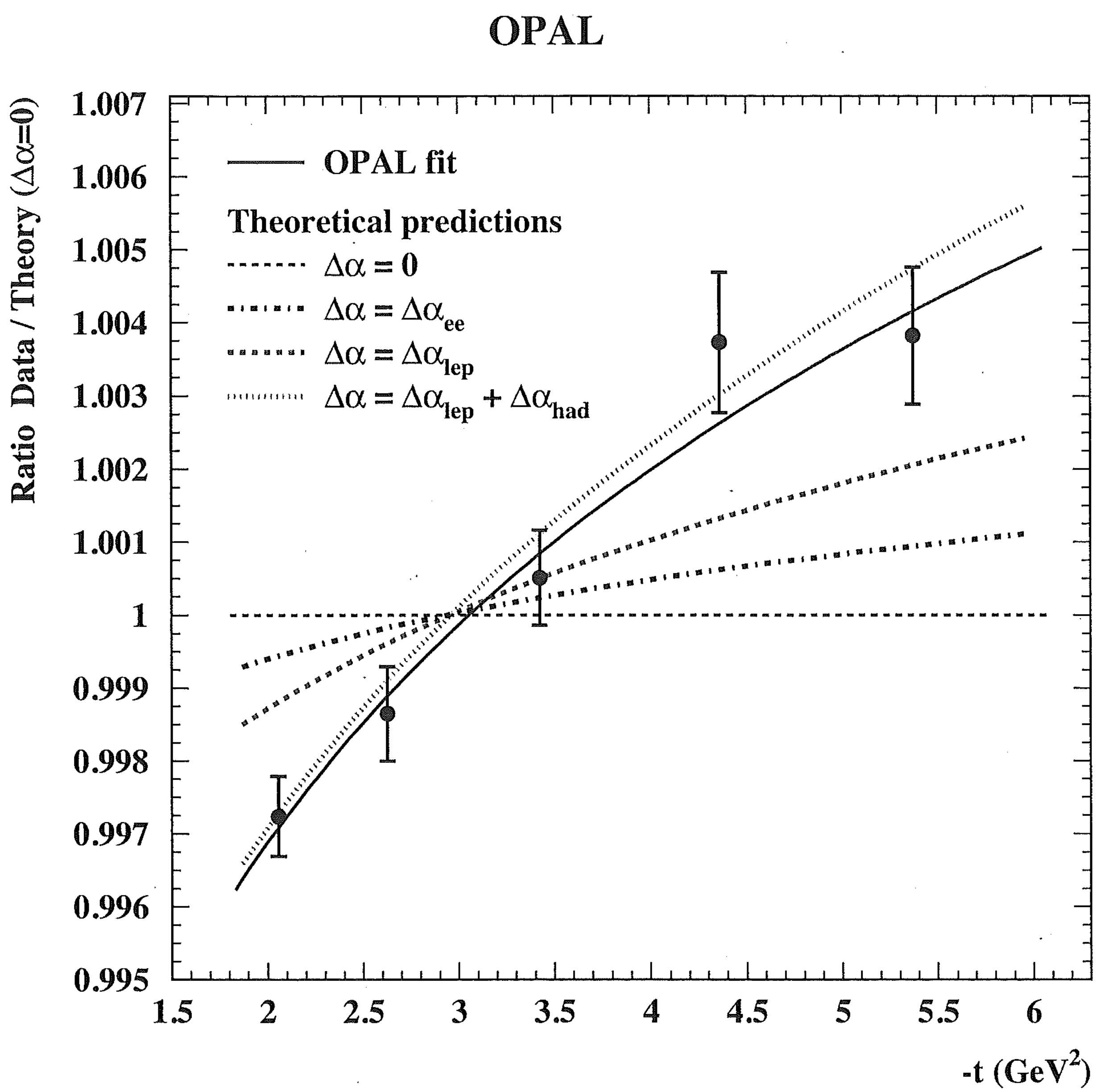
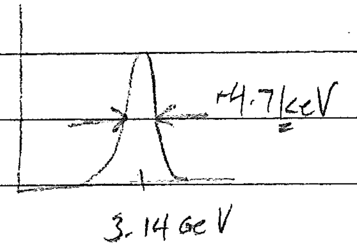


Figure 12: $|t|$ spectrum normalized to the BHLUMI theoretical prediction for a fixed coupling ($\Delta\alpha = 0$). The points show the combined OPAL data with statistical error bars. The solid line is our fit. The horizontal line (Ratio=1) is the prediction if α were fixed. The dot-dashed curve is the prediction of running α determined by vacuum polarization with only virtual e^+e^- pairs, the dashed curve includes all charged lepton pairs and the dotted curve the full Standard Model prediction, with both lepton and quark pairs.

I. Asymptotic freedom and the J/ψ

The c quark was discovered when accelerator energy was scanned across the threshold for producing 2 c's (i.e. $c\bar{c}$), they were produced and immediately bound. Call this "charmonium"

Noteworthy: extremely narrow resonance



Ground state is called η_c. ~~Next state is~~ J/ψ

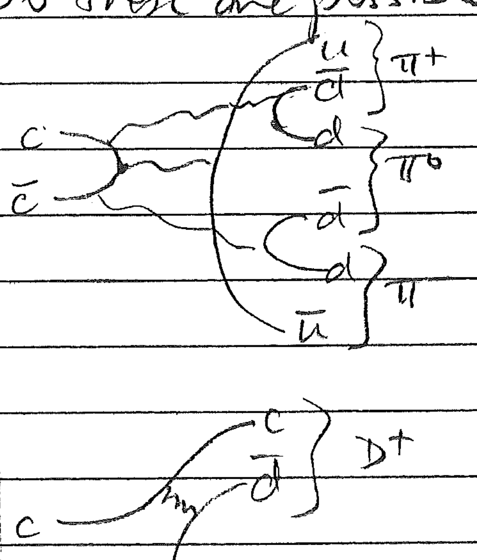
Small $\Delta E \rightarrow$ Large $\Delta t \rightarrow$ long lifetime. Why?
Quarks bind in shells analogous to electron orbitals.

Lowest E states of $c\bar{c}$: $^1S_0$ and $^3S_1$

	$^1S_0$	$^3S_1$
Charge conjugation eigenvalue C =	-1	$+1$
	requires 2-body final state	requires 3-body final state

a feature of the wavefunction that is conserved by strong force, like P, E, L, etc

So these are possible:



Only allowed.
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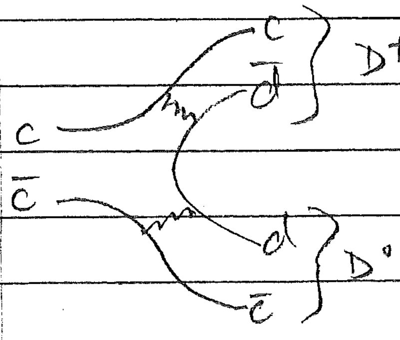


Forbidden by energy conservation:
$m(D) > m(J/\psi)$

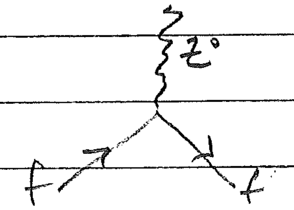
Allowed decay requires simultaneous production of 3 gluons which each carry high $E \sim m_\pi \approx 140$ MeV.

high energy → weak α_s → interaction less frequent ("suppressed")
So lifetime is long.

II Weak interaction

Basic vertices:

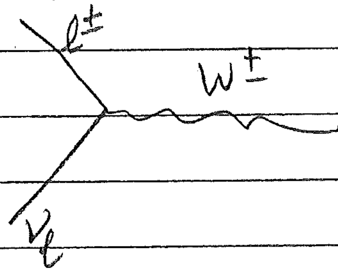
(i)



f = fermion (any spin ½)
all q, all l

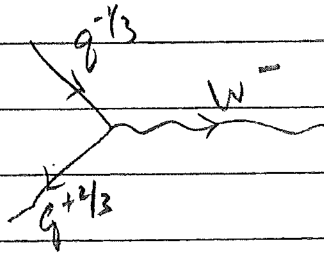
Note this means that all diagrams that are possible for the γ are also possible for the Z.
Probability ~ $|\sum \text{Amplitudes}|^2$ can include γ-Z interference effects.

(ii)

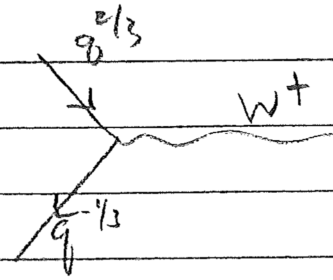


Charge must be conserved at all Feynman diagram vertices

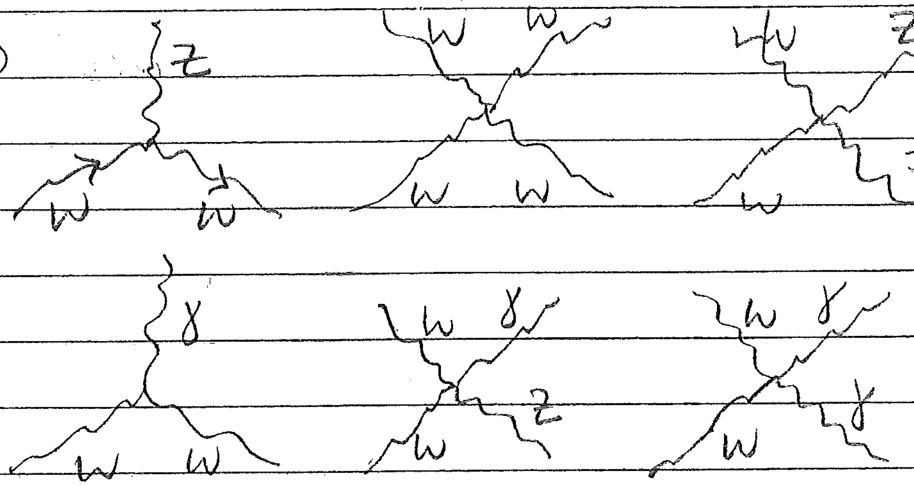
(iii)



or



(iv)

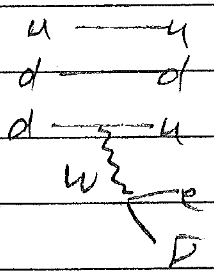


Neutron decay

a weak process — both typical + unusual

$n \to p + e + \bar{\nu}_e$ $\tau \sim 15$ minutes [free n]

$\quad\quad\quad\quad\quad\quad\quad\quad\quad$ " $\to \infty$ [bound n]



Why?

$m_{\text{Free } p} = 938.27$ MeV

$m_{\text{Free } n} = 939.56$ MeV

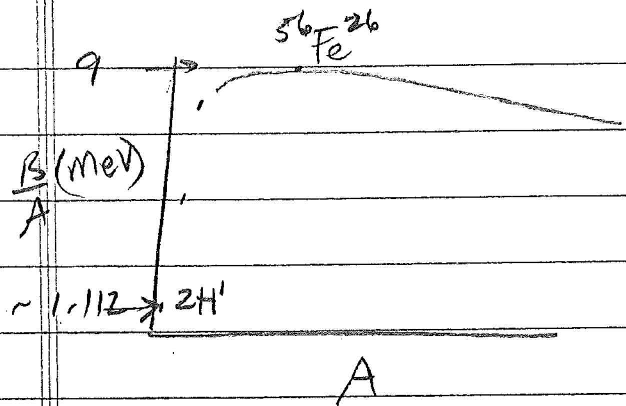
Binding energy per nucleon in a nucleus is

$$\frac{\text{"}B\text{"}}{A} = \frac{-\text{binding energy}}{\text{atomic \#}} = \frac{-(\Delta M) c^2}{A}$$

mass deficit or "defect"

$m_{\text{nucleus}} - \sum m_{\text{nucleons}}$

Empirically is



B/n

$m_{\text{Deuteron}} = 1875.6$ MeV

$m_{\text{Deuteron constituents}} = \underbrace{2(938.27)}_{p} + \underbrace{0.511}_{e} + \underbrace{0}_{\nu} = 1877.05$ MeV

Kinematically forbidden by 1.45 MeV

This accident of stability was essential for the formation of matter in the universe.

I Symmetry and Conservation

Def. symmetry :: Correspondence in form and configuration on opposite sides of a point

• A characteristic that is invariant to the application of an operation/transformation

Noether's Theorem:
"For every continuous transforma under which a Lagrangian is invariant, there exists a conserved current."

Ex: invariance of $\mathcal{L}$ wrt x (position) implies conservation of $\vec{p}$ (momentum)

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = 0$$

Assume force-free environment,
So $\mathcal{L} = T - U \to T$ only
because $F = -\nabla U$, so no $F \Rightarrow$ no U
Then $\mathcal{L} = \frac{p^2}{2m} \neq \mathcal{L}(x)$

$= \frac{m\dot{x}^2}{2}$

0

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} = 0$$

Note $\frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x} = p$

$$\frac{d}{dt}(p) = 0$$

p = constant

Known relationships between symmetries + currents

Sym	Conserved current
x (translation)	$\vec{P}$
θ (rotation)	L
t (time translation)	E
gauge	charge
B, L	?

gauge is the choice of 2 terms in an equation such that their effects offset each other, leaving the solution to the equation unchanged. Ex for Maxwell's Laws:

maxwell $\nabla \cdot B = 0$

vector calc. $\nabla \cdot (\nabla \times A) = 0$ (any A)

$\rightarrow B = \nabla \times A$

Maxwell $B = \nabla \times A$

vector calc $\nabla \times (\nabla \Lambda) = 0$ (any Λ)

$\rightarrow$ for any B and A_0, if $B = \nabla \times A_0$, then $B = \nabla \times (A_0 + \nabla \Lambda)$ also works, for any Λ

Choose any Λ that does not contradict Maxwell's other 2 equations:

maxwell $\nabla \times E = -\dfrac{dB}{dt}$

$B = \nabla \times A$

$\rightarrow \nabla \times E = -\dfrac{d}{dt}(\nabla \times A)$

$\nabla \times \left(E + \dfrac{dA}{dt}\right) = 0$

Maxwell $\nabla \times \left(E + \frac{\partial A}{\partial t} \right) = 0$

Vector calc $\nabla \times \nabla \Lambda = 0$, any Λ

$\rightarrow \quad E + \frac{\partial A}{\partial t} = -\nabla \varphi$

Name this $\Lambda = -\varphi$

$E + \frac{\partial A}{\partial t} = -\nabla \varphi$

$A \rightarrow A^{+} \equiv A + \nabla \Lambda$

$\longrightarrow \quad E + \frac{\partial}{\partial t}\left(A^{+} - \nabla \Lambda \right) = -\nabla \varphi$

$E + \frac{\partial A^{+}}{\partial t} = -\nabla \underbrace{\left(\varphi - \frac{\partial \Lambda}{\partial t} \right)}_{\text{call this } \varphi^{+}}$

Conclusion: Maxwell's Laws are unchanged by the simultaneous transformations

$A \rightarrow A + \nabla \Lambda$

$\varphi \rightarrow \varphi - \frac{\partial \Lambda}{\partial t}$

as long as these are the same Λ

The combined choice of A and φ is the "choice of gauge

This leads to conserved electrical current because the Maxwell Eqs can be combined to form the continuity eq:

$\nabla \times B = J + \frac{\partial E}{\partial t}$

Take $\nabla \cdot$ of both sides

$\underbrace{\nabla \cdot \nabla \times B}_{0} = \nabla \cdot J + \frac{\partial}{\partial t}(\nabla \cdot E)$

So $\nabla \cdot J + \frac{\partial}{\partial t}(\nabla \cdot E)$

But $\nabla \cdot E = \rho$

$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$

To match the language of Noether we could condense Maxwell's Eq. into Lagrange's Eq.:

$\mathcal{L} = \frac{1}{16\pi} g_{\lambda\mu} g_{\nu\sigma} (\partial^\mu A^\sigma - \partial^\sigma A^\mu)(\partial^\lambda A^\nu - \partial^\nu A^\lambda) - \frac{1}{c} J_\alpha A^\alpha$

I Using group theory to describe particle systems

Group theory - first applied to atomic physics ~1930
extended to particle physics ~1955

Consider a set of transformations that when applied to a system, leave it unchanged. Call these "symmetry operations". Examples in coord-space could be rotations, inversions, etc. Consider a particular family called R_i which satisfy:

① A product is defined: $R_k = R_i R_j$, and for every pair of members R_j, R_i, The product R_k is also a member.

- this is closure

"Product" means apply 2 operations sequentially; Not necessarily a math multiplication.

- The product can be generalized to other mathematical operations such as addition or sequential rotation

② The identity I is a member

- doing nothing is a symmetry operation

③ Every R_i has an inverse which is also a member of the group, so $R_i^{-1} R_i = I$

- inverse can be generalized to division, backward rotation, etc.

④ Application of the operations is associative:

$$R_i (R_j R_k) = (R_i R_j) R_k$$

→ This family constitutes a group.

Note commutivity is not required. - Order of application of the operations can be important. - relevant for physics, e.g. for strong interaction

Note the members of the group are the transformations (e.g. "rotate 60°") (e.g. "raise i to power 2") not the system being transformed

There are groups that have little relationships to physical systems (ex. - C_4, the (4 element) set of all powers of "i") "order 4" and some that are realized in the physical world.

e.g. SU(3)