Now you could have 2 points of view:

View #1

The principal of local gauge invariance generates the EM interaction

-OR-

View #2

The EM interaction exists and it manifests local gauge invariance as one of its properties.

Which view is more fundamental?

The principle of gauge invariance can ALSO

- predict charge conservation
- predict the masslessness of the photon

whereas View #2 (the description of the EM field) CANNOT.

So perhaps View #1 is primary? . . .

$\boxed{5.}$ Requiring local gauge invariance generates the strong force too!

We showed:

Allowing $\psi \rightarrow e^{i\theta(x)}\psi$ calls into being $\vec{A}$, the EM force.

We treated $\theta(x)$ as a scalar.

Generalize: let the $\theta(x)$ be matrices.

Suppose $\Theta(x)$ is 3×3 and unitary.

Expand it in a basis "T": $\Theta(x) = \sum_i \alpha_i T^i$

Again require $\psi(x) \to e^{i\Theta(x)}\psi(x)$ maintains the form of the Dirac Equation.

You are forced to create a new covariant derivative

$$\partial_\mu \to D_\mu \equiv \partial_\mu + i\underline{g}\, T_i\, \underline{G^i_\mu}$$

G_μ is a field like A_μ

g is a charge like e

The G^i_μ are the <u>gluons</u>! (there are 8 of them).

and this is the <u>strong force</u>!

It turns out that the T_i don't commute:

$$[T_i, T_j] = i f_{ijk} T_k .$$

We say the group that the T generate is "non-abelian"

The group is unitary, $\det = 1$ 3×3 matrices, is $SU(3)$)

An extra complication —

Recall that a gauge transformation requires that the derivative (∂_μ) and the field (A_μ or G_μ) transform together. The transformation of G_μ is different from the transformation of A_μ — that's good — it makes the strong force different from the electroweak.

Transformations:

E+M	Strong
$\partial_\mu \to D_\mu \equiv \partial_\mu + ieA_\mu$	$\partial_\mu \to D_\mu \equiv \partial_\mu + igT_a G_\mu$
$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \theta(x)$	$G_\mu^a \to G_\mu^a - \frac{1}{g}\partial_\mu \alpha_a - \underbrace{f_{abc}\,\alpha_b\, G_\mu^c}$

Because the T's are (non-commuting) matrices, you get this extra term.

What physics does the extra term $(-f_{abc}\,\alpha_b\,G_\mu{}^c)$ give to the strong force that is not present in the EW force?

$\downarrow$

Gluons can interact with each other; photons can't.

$\downarrow$

This produces asymptotic freedom, the property that the strong force becomes very weak at short distances, so the quarks + gluons are unbound inside the nucleus.

$\downarrow$

The strong coupling "constant" changes with distance.

$\downarrow$

How this works...

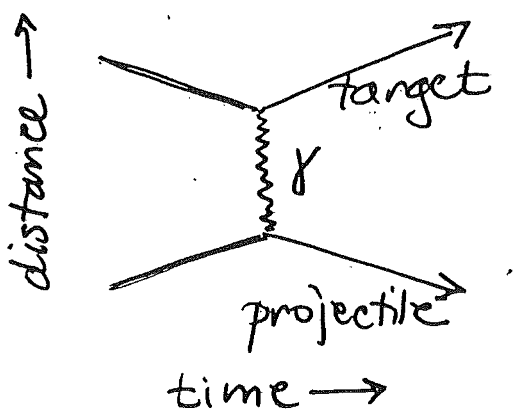
Suppose you want to study the strengths of the forces.

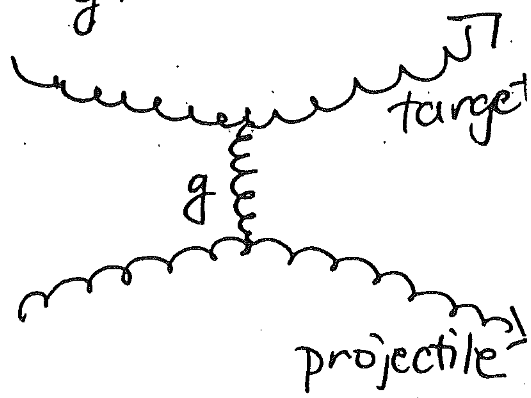
STRENGTH = magnitude of the coupling constant,

$\propto$ magnitude of the charge.

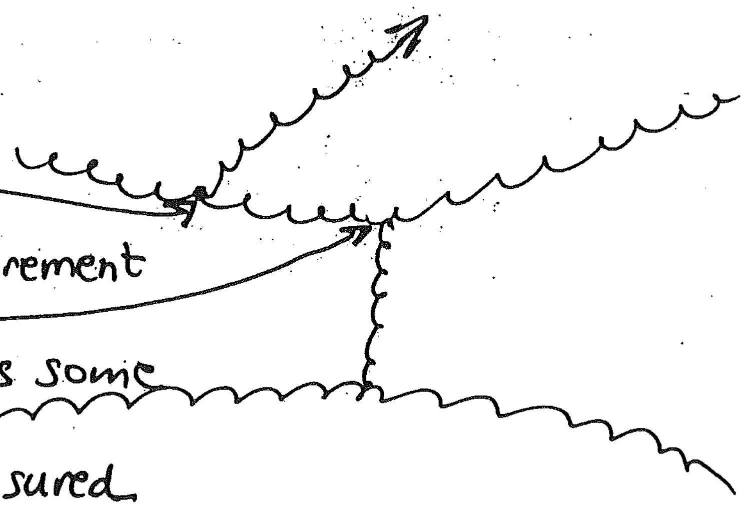
$\Rightarrow$ Indicates the probability that the particle under test will emit the kind of boson (photon, gluon, ...) that carries the force.

	EM force	Strong force
…ling …stant:	$\alpha \equiv \dfrac{e^2}{\hbar c}$	α_s
roportional to:	electric charge "e"	color charge "g"
ire a projectile t a target	electron electron	gluon gluon
this can happen:	The target emits photons:	The target emits gluons:



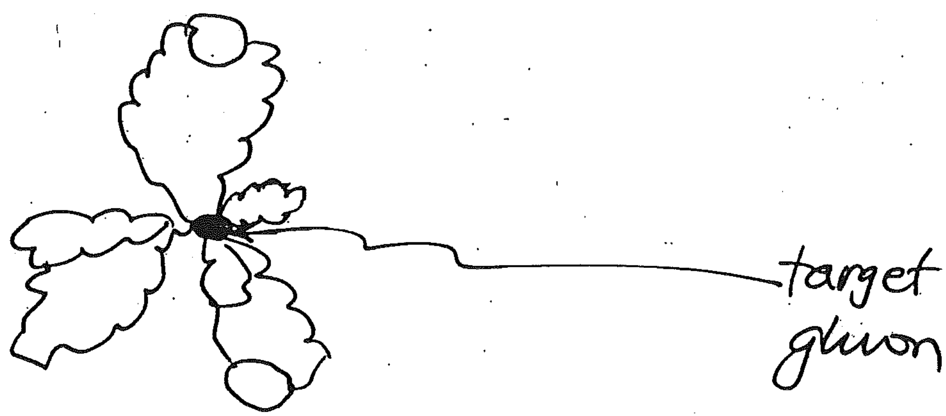


But also this can happen:

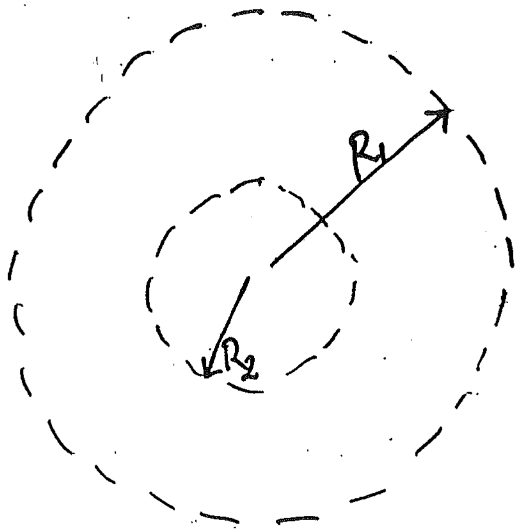


Before the measurement occurs here, the target radiates some of its color away here, so the measured strength appears weaker.

* The radiated gluons always eventually connect back to the target (* there are no free quarks/gluons), but the loops they make can be large:

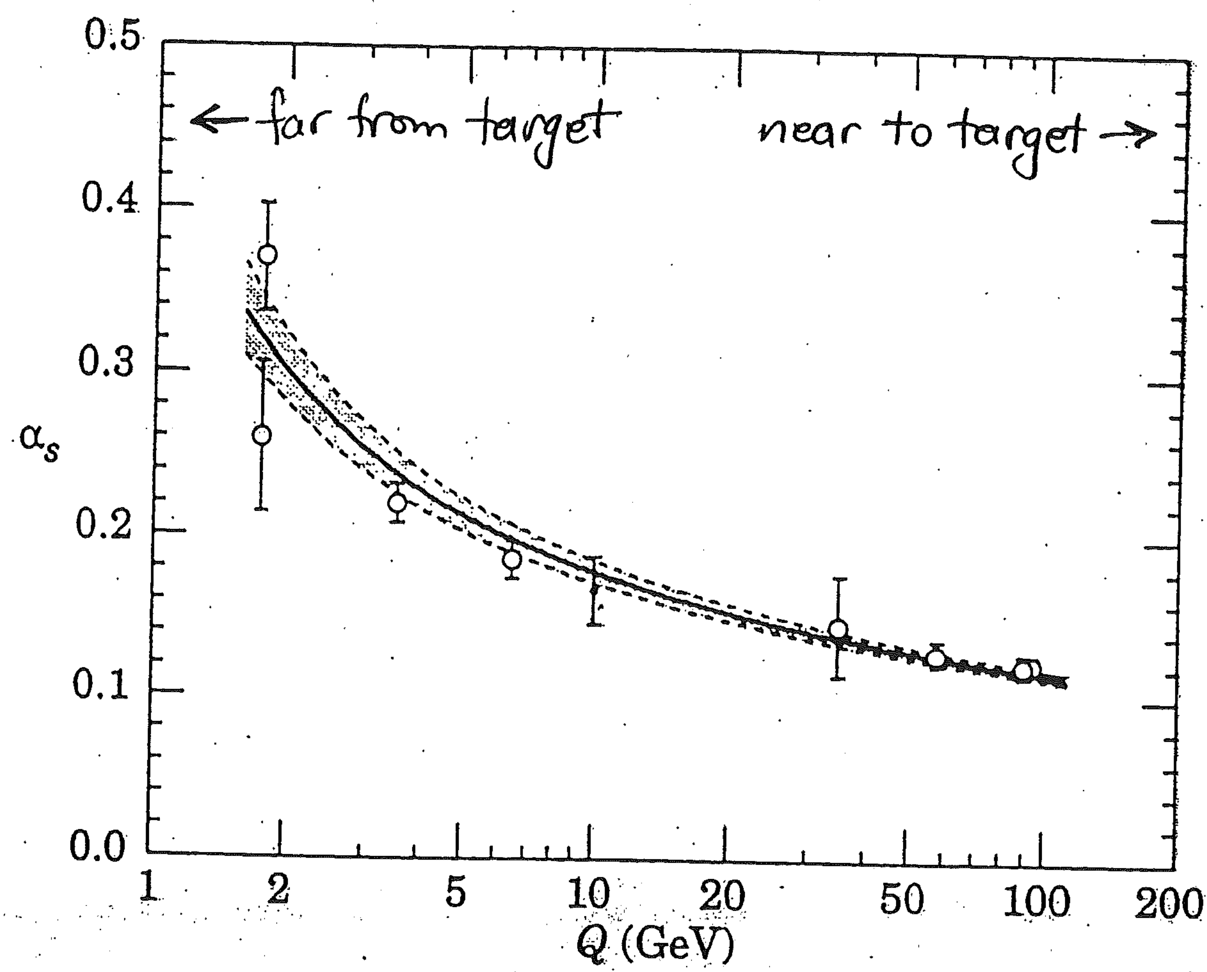


so the closer the projectile comes to the target, the more likely it is to "miss some of the color."



A projectile that recoils @ radius R_1 will sense all of the color; a projectile that gets closer (to within R_2) will miss some color.

The higher the projectile's energy, the closer it gets the target, so we expect α_s to decrease as momentum transfer Q increases. That's what we see:



Summary of the values of $\alpha_s(Q)$ at the values of Q where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average.

We say, "α_s runs with energy."

I Massive propagators

The main reason why local gauge invariance is taken as a starting point for particle theory is that it guarantees that the theory is renormalizable

Recall that local gauge invariance requires that we add to a theory of free fields ψ a new field

for example A_μ ("the gauge field" which is the propagator) and we allow A_μ to have the freedom (choice of gauge)

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda$$

Recall recipe for generating propagator formulas without considering Local Gauge Invariance (19 Sept lecture):

(1) Write Lagrangian (cooked up)

(2) Convert $\partial_\mu \rightarrow -ip_\mu$. Get something $\times$ field (e.g. $[\not{p} - m]\psi = 0$)

(3) Mult [ ] by $-i$ and take inverse.

For W/Z this lead to $$\frac{i\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\right)}{p^2 - m^2}$$

The Lagrangian that leads to this is

w/ Jackson p. 349

$$\mathcal{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{mc}{\hbar}\right)^2 A^\nu A_\nu$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

So we expect that the indication of a massive propagator is a term in the $\mathcal{L}$ of the form

$$-(\text{positive})(\text{parameter})^2 (\text{field})^2$$

$$+\left(\frac{mc}{\hbar}\right)^2 (A^\nu A_\nu)$$

Generically this would look like $+\mu^2 \phi^2$

Problem with how this propagator was constructed:

although $F^{\mu\nu}$ is invariant under gauge transf, $A^\nu A_\nu$ is not

So this $\mathcal{L}$ only satisfies gauge requirements if $m = 0$.

So propagators must be massless if they appear explicitly in the Lagrangian in this way.

If we know that the propagator really is massive, we must modify $\mathcal{L}$ to include a "$-\mu^2\phi^2$" term but also include other terms which taken as a whole make $\mathcal{L}$ locally gauge invariant.

Doing this is called:

Griffiths p. 361 II Spontaneous Symmetry Breaking

Consider some $\mathcal{L}$ for 2 fields ϕ_1 and ϕ_2:

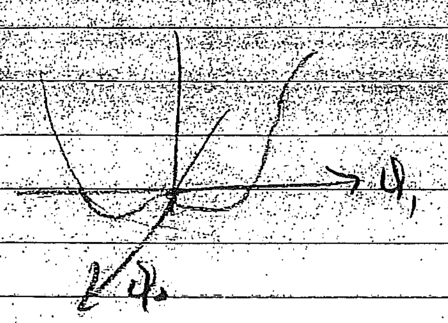
$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu\phi_1)(\partial^\mu\phi_1) + \tfrac{1}{2}(\partial_\mu\phi_2)(\partial^\mu\phi_2)}_{T} \underbrace{+\tfrac{1}{2}\mu^2(\phi_1^2+\phi_2^2) - \tfrac{1}{4}\lambda^2(\phi_1^2+\phi_2^2)^2}_{-U}$$

$$= T - U$$

This $\mathcal{L}$ is invariant to rotations in (ϕ_1, ϕ_2) space.

ie, we could transform $\phi_1 \to \phi_1\cos\theta + \phi_2\sin\theta$
$\phi_2 \to -\phi_1\sin\theta + \phi_2\cos\theta$

So the "U" part represents a circularly-symmetric well



How to identify the mass term:

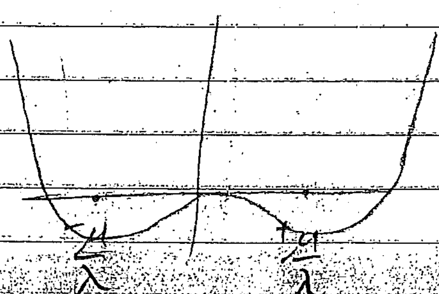
Feynman rules are derived by identifying specific terms in a perturbative expansion of $\mathcal{M}$.

Perturbation must be done about a minimum of the function, not just an inflection point.

So find the minimum:

$$U = -\frac{1}{2}\mu^2(\phi_1^2+\phi_2^2) + \frac{1}{4}\lambda^2(\phi_1^2+\phi_2^2)^2$$

Minima lie on the circle $\phi_{1min}^2 + \phi_{2min}^2 = \frac{\mu^2}{\lambda^2}$



Pick 1 example point on this circle:

$$\left(\phi_{1min} = \frac{+\mu}{\lambda},\ \phi_{2min} = 0\right)$$

Perturb fields by small amounts η and ξ about this point:

$$\phi_1 = \phi_{1min} + \eta \quad = \frac{\mu}{\lambda} + \eta$$

$$\phi_2 = \phi_{2min} + \xi \quad = 0 + \xi$$

Now
$$\mathcal{L} = \underbrace{\left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2 \eta^2\right]}_{\text{Term 1}} + \underbrace{\left[\frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)\right]}_{\text{Term 2}}$$
$$+ \underbrace{\left[\mu\lambda(\eta^3 + \eta\xi^2) - \frac{\lambda^2}{4}(\eta^4 + \xi^4 + 2\eta^2\xi^2)\right]}_{\text{Term 3}} + \frac{\mu^4}{4\lambda^2}$$

Compare this to the Klein Gordon Lagrangian:

$$\mathcal{L}_{KG} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 \phi^2$$

which describes a scalar (spin 0) particle ϕ of mass "m"

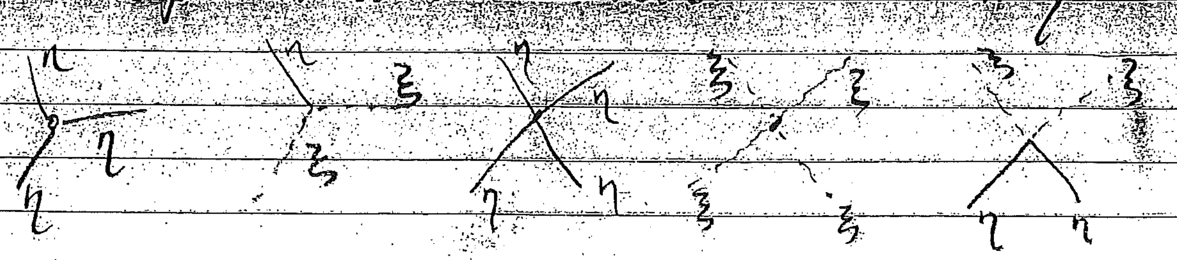
By comparison we see that

Term 1 describes a scalar particle η of mass $\frac{\sqrt{2}\cdot\hbar\mu}{c}$

Term 2 " " " " ξ mass 0

Term 2 ("The Goldstone Boson", a phantom that will go away)

Term 3 represents allowed interactions between η and ξ



Now combine φ_1 and φ_2 into a complex field

$$\varphi \equiv \varphi_1 + i\varphi_2$$

Then $\varphi_1^2 + \varphi_2^2 = \varphi^* \varphi$

So $\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^*(\partial^\mu \varphi) + \frac{1}{2}\mu^2(\varphi^*\varphi) - \frac{1}{4}\lambda^2(\varphi^*\varphi)^2$

Now require local gauge invariance $\varphi \to e^{i\theta(x)}\varphi$

which demands $\partial_\mu \to D_\mu \equiv \partial_\mu + \frac{iq}{\hbar c}A_\mu$

Then

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\varphi^*\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\varphi + \frac{1}{2}\mu^2\varphi^*\varphi - \frac{1}{4}\lambda^2(\varphi^*\varphi)^2$$

$$-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

(remember $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$)

So this $\mathcal{L}$ is guaranteed to be Locally Gauge Invariant

Return to the notation $\eta = \varphi_1 - \frac{\mu}{\lambda} \to \varphi_1 = \eta + \frac{\mu}{\lambda}$

$$\xi = \varphi_2 \qquad \varphi_2 = \xi$$

So $\varphi = \eta + \frac{\mu}{\lambda} + i\xi$

Then
$$\mathcal{L} = \overbrace{\left[\frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \mu^2\eta^2\right]}^{\text{Term 1}} + \overbrace{\left[\frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)\right]}^{\text{Term 2}}$$

$$+ \overbrace{\left[\frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)^2 A_\mu A^\mu\right]}^{\text{Term 3}} \overbrace{- 2i\left(\frac{\mu}{\lambda}\frac{q}{\hbar c}\right)(\partial_\mu\xi)A^\mu}^{\text{Term 4}}$$

— Term 5 —

$$+ \left\{ \frac{q}{\hbar c}\left[\eta(\partial_\mu\xi) - \xi(\partial_\mu\eta)\right]A^\mu + \frac{\mu}{\lambda}\left(\frac{q}{\hbar c}\right)^2 \eta(A_\mu A^\mu) \right.$$

$$+ \frac{1}{2}\left(\frac{q}{\hbar c}\right)^2(\xi^2+\eta^2)(A_\mu A^\mu) - \lambda\mu(\eta^3 + \eta\xi^2)$$

$$\left. - \frac{1}{4}\lambda^2(\eta^4 + 2\eta^2\xi^2 + \xi^4)\right\} + \left(\frac{\mu^2}{2\lambda}\right)^2$$

Term 1: still η field with mass $\sqrt{2}\,\frac{\mu\hbar}{c}$

Term 2: " ξ " " mass 0

Term 3: field A_μ now has mass $2\sqrt{\pi}\left(\frac{q\mu}{\lambda c^2}\right)$

Term 5: allowed couplings between ξ, η, A

Term 4: looks odd – indicates

Indicates that the wrong expressions are being identified with the physical fields. How to make it go away (This also removes ξ altogether):

Recall we can let $\phi \to \phi' = e^{i\theta(x)}\phi = \phi_1' + i\phi_2'$

$$= (\cos\theta + i\sin\theta)(\phi_1 + i\phi_2)$$

$$= [\phi_1\cos\theta - \phi_2\sin\theta] + i[\phi_1\sin\theta + \phi_2\cos\theta]$$

Choose $\theta = -\tan^{-1}\left(\frac{\phi_2}{\phi_1}\right)$

Then $\tan(-\theta) = \frac{\phi_2}{\phi_1}$

$$\tan\theta = -\frac{\phi_2}{\phi_1}$$

$$\sin\theta = -\phi_2 / (\phi_1^2 + \phi_2^2)$$

$$\cos\theta = \phi_1 / (\phi_1^2 + \phi_2^2)$$

Then $\underbrace{\mathrm{Im}\,\phi'}_{=\phi_2'} = \phi_1\left(\frac{-\phi_2}{\phi_1^2+\phi_2^2}\right) + \phi_2\left(\frac{\phi_1}{\phi_1^2+\phi_2^2}\right) = 0$

But $\phi_2' = \xi$

So ξ is gone

I The Higgs mechanism

The η particle above is the Higgs. It exists to give mass to other particles (...

This process of combining spontaneous symmetry breaking + local gauge inv. in order to produce a massive propagator is called the Higgs Mechanism

The mechanism transfers degrees of freedom:

$$\left.\begin{array}{l}\text{massless (vector) } A^{\mu}:\ 2 \text{ d.o.f.} \\ \text{massless (scalar) } \xi:\ 1 \text{ d.o.f.}\end{array}\right\} \rightarrow \text{massive } A^{\mu}:\ 3 \text{ d.o.f. only}$$

I. The Matter/Antimatter Asymmetry of The Universe

"Why is there something rather than nothing?"

Theoretically: 1) Dirac Eq. naturally predicts particle-antiparticle pairs.

Weinberg Sect 3.3 + 3.6

2) CPT invariance (needed for Lorentz Inv. + unitarity in any field theory) predicts identical decay rates, masses, etc., for the members of the pair

Experimentally: no violation observed in pair production

So one would expect that equal amts. of matter + antimatter were created in the Big Bang, and all should have annihilated by now, leaving nothing.

How did the matter asymmetry emerge?