Quantum Mechanics 2 Homework #1

1) Use separation of variables in Cartesian coordinates to solve the infinite 3-dimensional well:

$$V(x,y,z)=\begin{cases}0 & \text{if } x,\ y, \text{ and } z \text{ are all between 0 and a}\\ \infty & \text{otherwise}\end{cases}$$

(a) Find the eigenfunctions and eigenenergies

(b) Call the first six distinct energies E_1, E_2, ..., E_6. Determine the degeneracy of each of these energies (i.e., find the number of different states that share the same energy.)

2) How that any Hamiltonian of the form

$$H=\frac{p^2}{2m}+V(|r|),$$

where $|r|$ is the magnitude of the three-dimensional distance $\vec{r}$, commutes with all three components L_x, L_y, and L_z of the operator $\vec{L}$. Since we already know that L^2 and L_z commute, this result indicates that H, L_z, and L^2 are mutually compatible observables.

3) Formulate Heisenberg's Uncertainty Principle for three dimensions. Support your answer with commutators.

4) Show that

(a) $[x, f(p)]=i\hbar\dfrac{\partial f}{\partial p}$

(b) $[x,p^n]=i\hbar np^{n-1}$

Quantum Mechanics 2 Homework #2

1) Goswami problem 12.10.

2) At a time $t = 0$, a particle is in the superposition state: $\psi(\vec{r},0) = \dfrac{1}{2\pi^{3/2}} \sin(3x)\exp[i(5y+z)]$

(a) If the energy is measured at $t = 0$, what value will be found?

(b) What possible values of momentum (p_x, p_y, p_z) will measurement find?

(c) Given the above expression for $\psi(\vec{r},0)$, what is $\psi(\vec{r},t)$?

(d) If p is measured at $t = 0$ and the value $\vec{p} = \hbar(3\hat{x} + 5\hat{y} + \hat{z})$ is found, what is $\psi(\vec{r},t)$?

3) The relativistic analog of the Schroedinger Equation for a spin-zero (i.e., non-physical) electron is $\left(\dfrac{E}{\hbar c} + \dfrac{Ze^2}{\hbar c r}\right)\psi = -\nabla^2\psi + \left(\dfrac{mc}{\hbar}\right)^2 \psi$. Find the radial equation.

4) Consider a particle trapped in a cylindrical well of radius a. The well is unbounded in z. The particle is consequently subject to the condition: $V(\rho) = \begin{cases} 0 & \text{if } \rho < a \\ \infty & \text{if } \rho \geq a \end{cases}$

(a) Write down the Schroedinger Equation in cylindrical coordinates for this particle.

(b) Guess that the equation can be solved by a wavefunction of the form $\psi = R(\rho)\Phi(\phi)$. No conditions are applied in z.)

(c) Find equations for $R(\rho)$ and $\Phi(\phi)$.

(d) List the boundary conditions to which $R(\rho)$ and $\Phi(\phi)$ are subject.

(e) Solve the equations if they are familiar to you.

5) Goswami problem 12.A1.

Quantum Mechanics 2 Homework #3

1) Goswami problem 12.8.

2) Assume that the deuteron is a bound state with $\ell = 0$, and the potential is a square well of range $r = 2.8 \times 10^{-13}$ cm. Given that the binding energy is -2.18 MeV, find the depth of the potential. Here is a hint about how to do this: first convert distances and masses into units of the reduced mass μ, so that the range is given in units of $\hbar / \mu c$ and the binding energy in units of μc^2. Notice that the binding energy is quite close to zero, so you can expand the potential around that for which the binding energy is zero.

3) Write down the eigenvalue condition for a square well potential of range a and depth V_0, for $\ell = 1$. The only symbols allowed in the equation are μ, V_0, E, a, fundamental constants, and trigonometric functions.

4) Write down the three-dimensional Schroedinger Equation for the harmonic oscillator potential. Separate the equation in spherical coordinates. Use the power series method to solve the radial equation. Find the recursion relation for the coefficients and determine the allowed energies.

5) Find the matrix representations of the operators L_x, L_y, and L_z in the basis $\left|\ell, m_\ell\right\rangle$, when $\ell = 1$. Arrange the rows and columns of the matrices in order of decreasing m_ℓ value. By explicit multiplication of the matrices, show that $\left[L_x, L_y\right] = i\hbar L_z$.

Quantum Mechanics 2 Homework #4

1) Goswami problem 13.5.

2) Goswami problem 13.6.

3) An electron in the Coulomb field of a proton is in a state described by the wavefunction

$$\psi = \frac{1}{6}\left[4\psi_{100} + 3\psi_{211} - \psi_{210} + \sqrt{10}\psi_{21-1}\right].$$

(a) What is the expectation value of the energy?

(b) What is the expectation value of L^2?

(c) What is the expectation value of L_z?

4) Construct the spatial wavefunction $\psi(n = 4,\ \ell = 2,\ m = 1)$ for hydrogen. Express your answer as a function of $r,\ \theta,\ \phi$, and a_0 only. (That is, you may not use other variables such as ρ or z, functions such as the Y_ℓ^m or Laguerre polynomials, or constants other than π, e, and numerals.

5) Calculate the probability current $\vec{J}$ of a system whose Hamiltonian is given by Goswami Equation 14.7. Be sure that probability remains conserved.

Quantum Mechanics 2 Homework #5

1) Consider a positronium atom that consists of an electron and a positron (a positively charged electron) in a hydrogen-like bound state. No nucleus is present. Write down the Hamiltonian for this system in the presence of a constant external magnetic field. Show that (ignoring spin) this system experiences no Zeeman effect.

2) Consider a particle of mass M attached to a rigid massless rod of fixed length R whose other end is fixed at the origin. The rod is free to rotate about its fixed point.

(a) Write down the Hamiltonian for this system (hint: recall the rigid rotator of Chapter 11.)

(b) If the particle carries a charge Q and rotor is placed in a constant magnetic field $\vec{B}$, what is the modified Hamiltonian?

3) Consider a charged particle moving in a uniform magnetic field. If the field has a magnitude of 10^4 gauss, what type of radiation (x-rays? microwaves?) does the particle emit?

4) Goswami problem 14.6.

5) Consider a charged particle in a magnetic field $\vec{B} = (0,0,B)$ and an electric field $\vec{E} = (E,0,0)$. Choose $\vec{A} = (0, Bx, 0)$.

(a) Confirm that this choice of $\vec{A}$ is consistent with the given form of $\vec{E}$.

(b) Find the eigenvalues and eigenvectors for this system.

Quantum Mechanics 2 Homework #6

1) Find the equations of motion of the spin operators $S_x(t)$, $S_y(t)$, and $S_z(t)$ in the presence of a Hamiltonian given by $H = eg\vec{S}(t)\cdot\vec{B}/2\mu c$. ($\vec{B}$ is magnetic field, e is electric charge, μ is particle mass, c is the speed of light, and g is a unitless constant.) Use the fact that $\left[S_x(t),S_y(t)\right] = i\hbar S_z(t)$ and cyclically. Consider the case in which $\vec{B} = (0,0,B)$ and solve for $\vec{S}(t)$ in terms of $\vec{S}(t=0)$.

2) Goswami problem 15.2.

3) Use the angular momentum ladder operators, customized for spin, to find the matrix representations of the operators S_x, S_y, and S_z for a spin-3/2 particle.

4) Goswami problem 15.A10. Do only the first of the three operators given in part (b).

5) Consider the Hamiltonian for the one-dimensional infinite square well. The bottom of the well lies between $x = 0$ and $x = L$. The goal of this problem is to write the matrix representation for this Hamiltonian.

(a) Write down the eigenfunctions of this Hamiltonian.

(b) Are these eigenfunctions a valid basis set for representing this Hamiltonian? Explain why or why not.

(c) If your answer to part (b) is "no," write down a basis that is appropriate for this Hamiltonian.

(d) Arrange the basis functions that you have chosen (from part (b) or part (c)) as row and column headings of a matrix. Find the Hamiltonian matrix in terms of the basis. If the matrix does not come out diagonal, diagonalize it and indicate the basis in which it is diagonal. A basis in which a Hamiltonian is diagonal constitutes "the energy representation" (in contrast to the x-space or p-space representation.)

Quantum Mechanics 2 Homework #7

1) Derive the eigenspinors of S_y.

2) Goswami problem 15.4.

3) Goswami problem 15.8.

4) Goswami problem 15.9.

5) Goswami problem 15.10.

Quantum Mechanics 2 Homework #8

1) Goswami problem 15.12.

2) Recall that any two linearly independent spinors span the space of complex two-dimensional vectors.

(a) Write down the eigenspinors S_x, S_y, and S_z in the S_z basis. This amounts to 6 eigenspinors.

(b) Show that any generic spinor, $\begin{pmatrix} a \\ b \end{pmatrix}$, may be expressed as a linear combination of any one of these three pairs of eigenspinors.

3) Consider the matrix $A = \begin{pmatrix} \gamma & 0 & i\beta\gamma \\ 0 & 1 & 0 \\ -i\beta\gamma & 0 & \gamma \end{pmatrix}$. It can be diagonalize by a matrix U.

(a) Find U.

(b) Show that the eigenvectors comprising U are orthogonal.

(c) Carry out the matrix multiplication to verify that $U^{-1}AU$ is diagonal.

(d) Show that U is unitary by obtaining $U^{\dagger}$ and verifying by matrix multiplication that $UU^{\dagger} = 1$.

4) Consider an angular momentum = 1 system represented by a state vector

$\psi = \frac{1}{\sqrt{26}} \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$. What is the probability that a measurement of L_x yields the value 0?

5) Construct a 2×2 unitary matrix that has at least 2 imaginary elements.

Quantum Mechanics 2 Homework #9

1) Goswami problem 16.3. Plug in $a = D \cdot \xi$, $b = -A$, and $c = E_0$ (as those are defined in Section 16.2) and interpret the results.

2) Goswami problem 16.8.

3) In an NMR experiment, a sample of water displays resonant absorption when the frequency of the transverse magnetic field has a value of 42.3 MHz. If the parallel magnetic field B_0 is 10000 gauss, what is the value of g for the proton?

4) Goswami problem 18.2

5) Goswami problem 18.4.

Quantum Mechanics 2 Homework #10

1) Goswami problem 18.5.

2) Goswami problem 18.6.

3) Goswami problem 18.7.

4) Goswami problem 18.8.

5) Goswami problem 22.2.

Quantum Mechanics 2 Homework #11

1) A hydrogen atom is placed in an electric field $\vec{E}(t)$ that is spatially uniform and has the time dependence:

$\vec{E}(t) = 0$ for $t < 0$

$\vec{E}(t) = E_0 \exp(-\gamma t)\hat{z}$ for $t > 0$, where γ is a constant.

What is the probability, as $t \to \infty$, that the hydrogen atom which is initially in the ground state will make a transition to the $2p$ state?

2) Goswami problem 18.11.

3) Nuclei sometimes decay from excited states to less excited states by a process known as internal conversion, in which a $1s$ electron is emitted. The perturbation that causes this can be written as $H = -\sum_{i=1}^{Z} \frac{e^2}{|\vec{r} - \vec{r}_i|}$, where $\vec{r}$ is the electron coordinate and $\vec{r}_i$ are the proton coordinates. The density of states for the final state electron can be written as $\rho = \frac{d^3 n}{dp^3} = \frac{V}{(2\pi\hbar)^3}$, where V is some arbitrary volume and p is the electron's momentum.

(a) Write down the initial state. Normalize it over the same volume V.

(b) Write down the final state.

(c) Write down the matrix element.

(d) Write down the expression for the transition rate.

4) Goswami problem 22.7, part (a) only.

5) Goswami problem 22.9. Be sure to state what happens to the total cross section as $a \to \infty$, and indicate what this means physically.