In-class problem linked to lecture pages 303-320:

Find the volume v_c at which a Van der Waals liquid attains its critical point, in terms of the Van der Waals parameters a and b.

Find the critical temperature T_c.

Find the critical pressure p_c.

In-class 303-320

$$p = \frac{RT}{v-b} - \frac{a}{v^2} = RT(v-b)^{-1} - av^{-2}$$

@ T_c, $\frac{dp}{dv} = 0$ and $\frac{d^2p}{dv^2} = 0$

$$\frac{dp}{dv} = \frac{RT(-1)}{(v-b)^2} - \frac{(-2)a}{v^3} = 0 \Rightarrow \frac{2a}{v^3} - \frac{RT}{(v-b)^2} = 0$$

$$\frac{d^2p}{dv^2} = \frac{-RT(-2)}{(v-b)^3} + \frac{2a(-3)}{v^4} = 0 \Rightarrow \frac{2RT}{(v-b)^3} - \frac{6a}{v^4} = 0$$

To find v_c, eliminate RT

$$\frac{\cancel{2a}}{\cancel{v^3}}\cancel{(v-b)^2} = \frac{\cancel{6}^{\,3}a}{v^4}\frac{(v-b)^3}{\cancel{2}}$$

$$2 = \frac{3(v-b)}{v}$$

$2v = 3v - 3b$

$0 = v - 3b$

$3b = v$

To find T_c, substitute $v = 3b$

$$\frac{2a}{3^3b^3} - \frac{RT_c}{(3b-b)^2} = 0$$

$$T_c = \frac{2a}{27\cancel{b^3}} \cdot \frac{1}{R} \cdot 4\cancel{b^2}$$

$$= \frac{8}{27} \frac{a}{bR}$$

To find p_c, substitute into $p = \frac{RT}{v-b} - \frac{a}{v^2}$

$$p_c = \frac{\cancel{R}\left(\frac{8a}{27b\cancel{R}}\right)}{3b - b} - \frac{a}{3^2b^2}$$

$$= \frac{\frac{8a}{27b}}{2b} - \frac{a}{9b^2}$$

$$= \frac{4a}{27b^2} - \frac{3a}{27b^2} = \frac{a}{27b^2}$$