In-class problem linked to lecture pages 281-294:

Use the partition function to find the average internal energy of a monatomic gas.

In-class problem linked to 281 -294

$$Z = \frac{V^N}{N!}\left(\frac{2\pi m kT}{h^2}\right)^{\frac{3N}{2}} = \frac{V^N}{N!}\left(\frac{2\pi m k}{h^2}\right)^{3N/2} T^{3N/2}$$

$$\ln Z = \ln\left[\frac{V^N}{N!}\left(\frac{2\pi m k}{h^2}\right)^{\frac{3N}{2}}\right] + \ln\left(T^{\frac{3N}{2}}\right)$$

$$\overline{E} = -\frac{\partial}{\partial\beta}\ln Z$$

Recall $\frac{d}{dx} f(y) = \frac{d}{dy} f(y) \cdot \frac{dy}{dx}$

$$\beta = \frac{1}{kT} \Rightarrow T = \frac{1}{k}\beta^{-1}$$

$$\frac{\partial}{\partial\beta} f(T) = \frac{\partial}{\partial T} f(T) \cdot \frac{\partial T}{\partial\beta}$$

$$\frac{\partial T}{\partial\beta} = \frac{1}{k}(-1)\beta^{-2} = -\frac{1}{k\beta^2}$$

$$\frac{\partial}{\partial\beta} = \frac{-1}{k\beta^2}\frac{\partial}{\partial T}$$

$$\overline{E} = -\left(\frac{-1}{k\beta^2}\right)\frac{\partial}{\partial T}\ln Z$$

$$= \frac{+1 k^2 T^2}{k}\frac{\partial}{\partial T}\left(\ln T^{\frac{3N}{2}}\right)$$

$$= kT^2\frac{\partial}{\partial T}\left(\frac{3N}{2}\ln T\right) = \frac{kT^2 3N}{2}\cdot\frac{1}{T} = \frac{3NkT}{2}$$