In-class problem linked to lecture pages 267-280

Consider a system whose energy levels are given by

$$E_s = f_s(T) - NkT \ln \frac{V}{V_0}.$$

Write the partition function for this system, and use it to determine the average pressure and chemical potential.

In class problem limited to Reine pages 267-280

$$E_s = f_s(T) - NkT \ln\left(\frac{V}{V_0}\right)$$

$$Z = \sum_s \exp(-\beta E_s)$$

$$= \sum_s \exp\left(-\beta f_s(T) + \cancel{\beta} N \cancel{kT} \ln\left(\frac{V}{V_0}\right)\right)$$

$$= \sum_s \exp(-\beta f_s(T)) \exp\left(N \ln \frac{V}{V_0}\right)$$

$\beta = \frac{1}{kT}$

$$= \underbrace{\exp\left(N \ln \frac{V}{V_0}\right)}\sum_s \exp(-\beta f_s(T))$$

$$= \left(\frac{V}{V_0}\right)^N \sum_s \exp(-\beta f_s(T))$$

$$\bar{P} = \frac{1}{\beta}\frac{\partial}{\partial V}\ln Z\bigg|_{T,N}$$

$$= \frac{1}{\beta}\frac{\partial}{\partial V}\ln\left[\left(\frac{V}{V_0}\right)^N \sum_s e^{-\beta f_s(T)}\right]$$

$$= \frac{1}{\beta}\frac{\partial}{\partial V}\left\{\ln\left(\frac{V^N}{V_0^N}\right) + \ln\sum_s e^{-\beta f_s(T)}\right\}$$

$$= \frac{1}{\beta}\frac{\partial}{\partial V}\left\{\ln(V^N) - \ln(V_0^N) + \ln\sum_s e^{-\beta f_s(T)}\right\}$$

$$= kT\frac{1}{V^N} N V^{N-1} = \underline{\frac{kTN}{V}}$$

$$\bar{\mu} = -\frac{1}{\beta}\frac{\partial}{\partial N}\ln Z\Big|_{TV}$$

$$= -\frac{1}{\beta}\frac{d}{dN}\left\{\ln V^N - \ln V_0^N + \ln\sum_s e^{-\beta f_s(T)}\right\}$$

$$= \frac{1}{\beta}\left\{\frac{1}{V^N}\frac{d}{dN}\left(V^N\right) - \frac{1}{V_0^N}\frac{d}{dN}\left(V_0^N\right)\right\}$$

Notice $V^N = \left(e^{\ln V}\right)^N = e^{N\ln V} = e^{(\ln V)N}$

So $\frac{d}{dN}e^{(\ln V)N} = \ln V\, e^{(\ln V)N} = (\ln V)\, V^N$

$$\bar{\mu} = -\frac{1}{\beta}\left\{\frac{1}{\cancel{V^N}}\ln V\cdot V^N - \frac{1}{V_0^N}\ln V_0\, V_0^N\right\}$$

$$= -\frac{1}{\beta}\ln\left(\frac{V}{V_0}\right)$$