In-class problem linked to lecture pages 219-231

The atomic mass of a single nitrogen atom is 14, and the separation between the two atoms in a nitrogen molecule is 3.15×10^{-10} m. The first excited rotational state has angular momentum $L = \sqrt{2}\hbar$. Calculate the moment of inertia of a nitrogen molecule about an axis perpendicular to the line joining the centers of the two atoms. Use your result to find the temperature at which the ratio of nitrogen atoms in the first excited state to those in the ground state is 1/e.

In-class 219 - 231

$\# \text{nucleons} = 14$

$L = 3.15 \times 10^{-10}\,m$

$$I = \frac{mL^2}{2}$$

m of proton $= 1.67 \times 10^{-27}\,kg$

$$I = \frac{14}{2}(1.67 \times 10^{-27}\,kg)(3.15 \times 10^{-10})^2\,m^2 =$$

$$= 1.16 \times 10^{-45}\,kg\,m^2$$

$$E_{rot} = \frac{L^2}{2I} = \frac{2\hbar^2}{2I} = \frac{\hbar^2}{I} =$$

$$\frac{P_{l=1}}{P_{l=0}} = \frac{e^{-\beta(\hbar^2/I)}}{e^{-0}} = e^{-\beta\hbar^2/I} \stackrel{?}{=} e^{-1}$$

i.e. $\frac{\beta\hbar^2}{I} = 1$

$$\frac{\hbar^2}{kTI} = 1$$

$$T = \frac{\hbar^2}{kI} = \frac{(1.06 \times 10^{-34})^2}{(1.381 \times 10^{-23})(1.14 \times 10^{-45})} = 0.7\,K$$