In-class problem linked to lecture pages 192-203

a) Find the temperature change associated with the free expansion of an ideal gas.

b) Find a formula for the temperature change associated with the free expansion of a van der Waals gas, in terms of pressure p, molar volume v, number of degrees of freedom υ, and the van der Waals coefficients a and b.

c) Consider a mole of liquid expanding freely from a volume of one liter to a volume twice that size. Each of its molecules has $\upsilon = 6$ degrees of freedom. Its van der Waals coefficients are

$a = \left(5.5 \text{ liters}^2 - \text{atm}\right)/\text{mole}^2$

$b = 0.03$ liters/mole.

What is its change in temperature?

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$$dT = \frac{1}{C_v}\left[p - T\left.\frac{dp}{dT}\right|_v\right]dV$$

a) Ideal gas: $pV = NkT$

$$p = \frac{NkT}{V}$$

$$\frac{dp}{dT} = \frac{Nk}{V}$$

$$\left[p - T\left.\frac{dp}{dT}\right|_v\right] = p - T\cdot\frac{Nk}{V} = p - p = \boxed{0}$$

b) Non-ideal: $\left(p + \frac{a}{v^2}\right)(v-b) = RT$

$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

$v = \frac{\text{Volume}}{\text{mole}}$

$$\left.\frac{dp}{dT}\right|_v = \frac{R}{v-b}$$

$R = N_A k$

$$\therefore \left.T\frac{dp}{dT}\right|_v = \frac{RT}{v-b} = p + \frac{a}{v^2}$$

So $p - T\frac{dp}{dT} = -\frac{a}{v^2}$

$$dT = \frac{1}{C_v}\left[p - T\left.\frac{dp}{dT}\right|_v\right]dV = \boxed{\frac{1}{C_v}\cdot\left(\frac{-a}{v^2}\right)dV}$$

c) Note $C_V = \frac{\nu N k}{2} = \frac{\nu R}{2}$

$dV \to dv$

$$\Delta T = \frac{-a \cdot 2}{\nu R} \int \frac{dv}{v^2}$$

$$\frac{-1}{v}\Big|_{\ell}^{2\ell}$$

$$= \frac{+2a}{\nu R}\left[\frac{-1}{2\ell} + \frac{1}{\ell}\right]$$

$$= \frac{-2a}{\nu R}\left[\frac{1}{2\ell}\right]$$

$$= \frac{-a}{\nu R}$$

$$= \frac{-5.5}{6 \cdot 8.21 \times 10^{-2}} = \boxed{-11\ K}$$