In-class problem linked to lecture pages 1-9:

Consider one molecule among a gas of diatomic molecules (such as nitrogen, N_2). Define a coordinate system such that the y-axis lies along the bond joining the two pointlike atoms, and the x- and z-axes are mutually perpendicular to this. The molecule can translate along the x, y, and z directions and can rotate about the x or z axes.

The temperature of the gas is $T = 300^\circ K$. The force holding the molecule together is F. Rotation about each axis contributes to the internal energy an amount $\frac{1}{2}kT$. Each atom has mass m. Find the average kinetic energy of a molecule in this gas using the information above.

Physics 301

In-class problem linked to pages 1-9

$\langle KE \rangle = \langle U \rangle$

$\vec{F} = -\vec{\nabla} U \approx \frac{U}{d/2} = \frac{2U}{d}$

So $\langle U \rangle \approx \langle \frac{F \cdot d}{2} \rangle$

We need $F \cdot d$

At equilibrium the binding force = the centrifugal force

$F = \frac{2m\langle v^2 \rangle}{r} = 2mr\langle \omega^2 \rangle$

Let $r = \frac{d}{2}$

To find $\langle \omega^2 \rangle$, note:

$\frac{1}{2} I \langle \omega^2 \rangle = \frac{1}{2} kT$, so $\langle \omega^2 \rangle = \frac{kT}{I}$

$I = 2mr^2 = 2m\left(\frac{d}{2}\right)^2 = \frac{md^2}{2}$

So $\langle \omega^2 \rangle = \frac{kT \cdot 2}{md^2}$

Then $F = 2m\left(\frac{d}{2}\right) \cdot \frac{kT \cdot 2}{md^2} = \frac{2kT}{d}$

So $Fd = 2kT$

$$\langle U \rangle = \frac{1}{2}\langle F \cdot d \rangle = kT$$

So $\langle KE \rangle = \langle U \rangle = kT$