Physics 301

Homework due 26 October 2022

1) Stowe Chapter 13 Section C includes a variety of examples of problems that use Maxwell's Relations. Invent another example that uses at least 2 of the relations. You may, but are not required to, base your example on one of the Stowe examples.

2) Stowe problem 13.12.

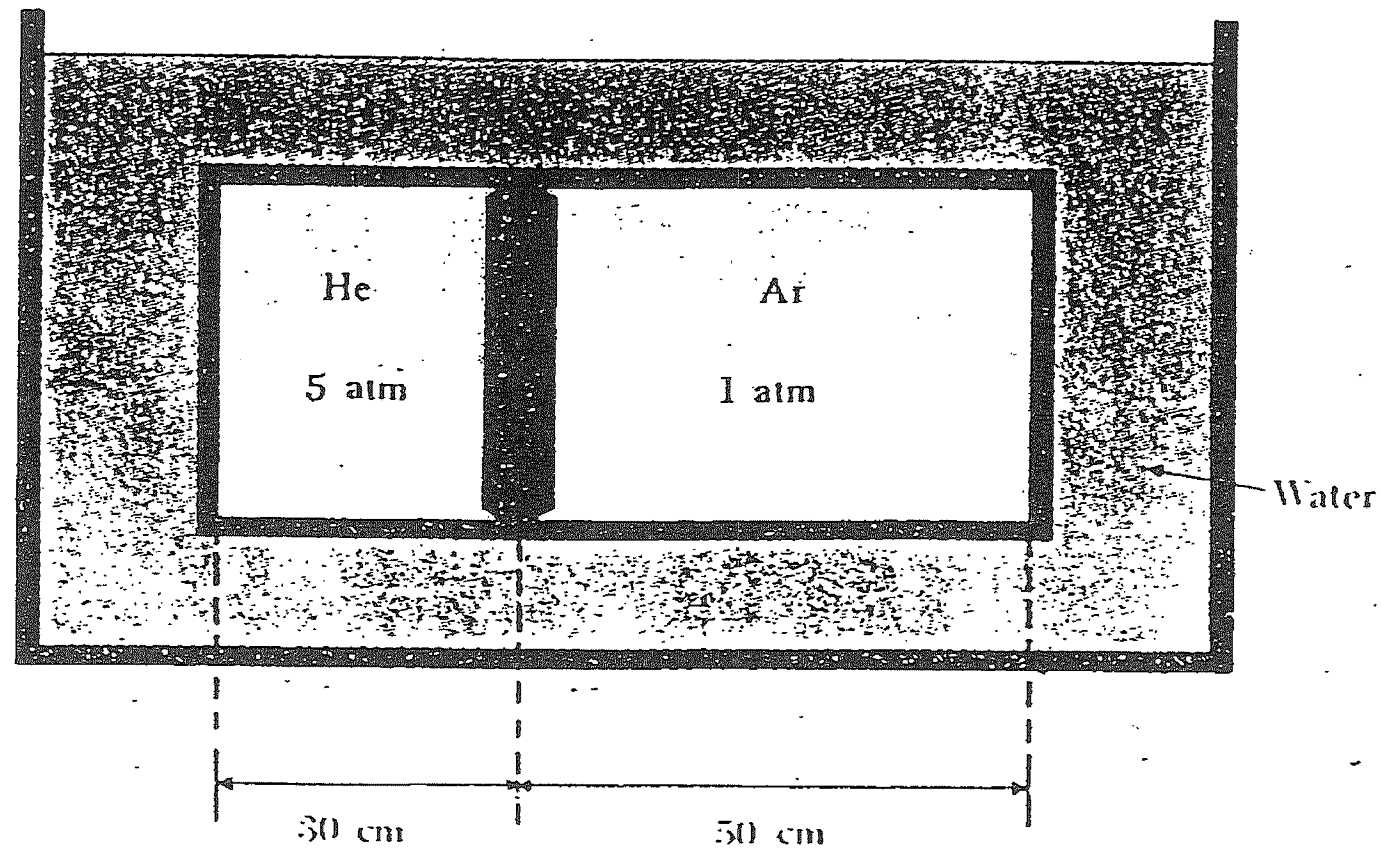
3) Stowe problem 14.6.

4) Stowe problem 14.9

5) (See attached figure.) A cylindrical container 80 cm long is separated into two compartments by a thin piston, initially clamped into position 30 cm from the left end. The left compartment is filled with 1 mole of helium gas at a pressure of 5 atm. The right compartment is filled with argon gas at pressure 1 atm. Both of these gases may be treated as ideal. The cylinder is submerged in 1 liter of water, and the entire system is initially at the uniform temperature of $25^{\circ}C$. The heat capacities of the cylinder and piston may be neglected. After the piston is unclamped, a new equilibrium is reached with the piston in a new position.

(a) What is the increase in temperature of the water?

(b) How far from the left end of the cylinder will the piston come to rest?


He
5 atm
Ar
1 atm
Water
30 cm
50 cm

Physics 301

Answers to homework due 26 Oct 2022

(2) $\Delta E = T\Delta S - p\Delta V$

a) Seeking $V(E,S)$

$$\Delta V = \frac{T\Delta S}{p} - \frac{1}{p}\Delta E$$

b) Seeking $V(p,S)$

$$\Delta V = \frac{T}{p}\Delta S - \frac{1}{p}\left(\left.\frac{\partial E}{\partial S}\right|_p \Delta S + \left.\frac{\partial E}{\partial p}\right|_S \Delta p\right)$$

$$= \frac{1}{p}\left(T - \left.\frac{\partial E}{\partial S}\right|_p\right)\Delta S - \frac{1}{p}\left.\frac{\partial E}{\partial p}\right|_S \Delta p$$

c) Seeking $V(p,T)$

$$\Delta V = \frac{T}{p}\left(\frac{\partial S}{\partial p}\Delta p + \frac{\partial S}{\partial T}\Delta T\right) - \frac{1}{p}\left(\frac{\partial E}{\partial p}\Delta p + \frac{\partial E}{\partial T}\Delta T\right)$$

$$= \frac{1}{p}\left[\left(T\left.\frac{\partial S}{\partial p}\right|_T - \left.\frac{\partial E}{\partial p}\right|_T\right)\Delta p + \left(\underbrace{T\left.\frac{\partial S}{\partial T}\right|_p}_{\text{This is also } C_p} - \left.\frac{\partial E}{\partial T}\right|_p\right)\Delta T\right]$$

d) Seeking $S(p,T)$

$$\Delta S = \frac{1}{T}\Delta E + \frac{p}{T}\Delta V$$

$$\Delta S = \frac{1}{T}\left(\frac{\partial E}{\partial p}\Delta p + \frac{\partial E}{\partial T}\Delta T\right) + \frac{p}{T}\left(\frac{\partial V}{\partial p}\Delta p + \frac{\partial V}{\partial T}\Delta T\right)$$

$$= \frac{1}{T}\left[\left(\left.\frac{\partial E}{\partial p}\right|_T + p\underbrace{\left.\frac{\partial V}{\partial p}\right|_T}_{\text{this is } V\kappa}\right)\Delta p + \left(\left.\frac{\partial E}{\partial T}\right|_p + p\underbrace{\left.\frac{\partial V}{\partial T}\right|_p}_{\text{this is } V\alpha}\right)\Delta T\right]$$

e) Could exchange for

$\left.\frac{\partial S}{\partial p}\right|_T$ (M10)

and

$\left.\frac{\partial V}{\partial T}\right|_p$ (M10)

3. pV^γ = constant for adiabatic processes in ideal gases.

This means

$$p_i V_i^\gamma = p_f V_f^\gamma \qquad \text{where } \gamma = \frac{\nu+2}{\nu}$$

$$\left(\frac{V_f}{V_i}\right)^\gamma = \frac{p_i}{p_f} \qquad \text{Suppose } V_f = V_i - \Delta V$$

$$\downarrow$$

$$\left(\frac{V_i - \Delta V}{V_i}\right)^\gamma = \frac{p_i}{p_f}$$

$$\downarrow \qquad \text{Let } \frac{-\Delta V}{V_i} = x \text{, a small parameter}$$

$$(1+x)^\gamma = \frac{p_i}{p_f}$$

$$\downarrow \qquad \text{Use } (1+x)^\gamma \approx 1 + \gamma x$$

$$1 + \gamma x \approx \frac{p_i}{p_f}$$

$$\downarrow$$

$$\gamma \approx \left(\frac{p_i}{p_f} - 1\right)\frac{V_i}{(V_f - V_i)} = \left(\frac{1}{1.1} - 1\right)\left(\frac{2}{1.85 - 2}\right) = 1.212 = 1 + \frac{2}{\nu}$$

$$\text{So } \nu = \frac{2}{1.212 - 1} = \boxed{9.4}$$

(4) $\left(p + \frac{a}{v^2}\right)(v-b) = RT$

$a = 5.5 \frac{l^2\text{-atm}}{\text{mole}^2}$

$b = 0.03 \frac{l}{\text{mole}}$

$p_1 = 100$ atm
$v_1 = 0.3 \, l$
moles = 1
$v_2 = 0.6 \, l$

a) $T_i = ?$

$$T = \frac{1}{R}\left(p + \frac{a}{v^2}\right)(v-b)$$

$$= \frac{1}{8.21 \times 10^{-2}}\left(100 + \frac{5.5}{(0.3)^2}\right)(0.3 - 0.03) =$$

$$= \boxed{530 K}$$

b) Isobaric: pressure is held constant

$$T_f = \frac{1}{8.21 \times 10^{-2}}\left(100 + \frac{5.5}{(0.6)^2}\right)(0.6 - 0.03) =$$

$$= \boxed{800 K}$$

c) ~~Isothermal~~: temperature is held constant

$$P = \left(\frac{RT}{v-b}\right) - \frac{a}{v^2}$$

$$= \frac{(8.21 \times 10^{-2})(530)}{(0.6 - 0.03)} - \frac{5.5}{(0.6)^2} = \boxed{61 \text{ atm}}$$

(5) $pV = NkT$ describes both gases. $T_i = 25°C = 298.15K$

At equilibrium, $p_{He} = p_{Ar}$

We are told that 1 mole of He is present. We do not kn

how much Ar is present.

We do not know the total volume of either gas (i.e., w

do not know the radius of the cylinder.)

Consider the Helium:

$$pV = NkT$$
$$\downarrow$$

$$V_{He} = \frac{N_{He}kT}{p_{He}} = \frac{(1\,\text{mole})\times\left(6.02\times10^{23}\,\frac{}{\text{mole}}\right)\left(1.381\times10^{-23}\,\frac{J}{K}\right)(298.15}{(5\,\text{atm})\left(\frac{1.013\times10^{5}\,N/m^2}{\text{atm}}\right)}$$

$$= 4.9\times10^{-3}\,m^3$$

So the radius of the cylinder $= \left(\frac{V_{He}}{\pi\cdot 0.3\,m}\right)^{1/2} = \left(\frac{4.9\times10^{-3}}{\pi\cdot(0.3)}\right)^{1/2}$

$$= 0.072 = \text{"}r\text{"}$$

Consider the Argon:

$$pV = NkT$$
$$\downarrow$$

$$N_{Ar} = \frac{p_{Ar}V_{Ar}}{kT_{Ar}} = \frac{(1\,\text{atm})\left(\frac{1.013\times10^{5}\,N/m^2}{\text{atm}}\right)(0.5\,m)\pi\cdot(0.07^2\,m^2)}{\left(1.381\times10^{-23}\,\frac{J}{K}\right)(298.15\,K)} = \boxed{1.89\times1}$$

At the new equilibrium point,

$$P_{He} = P_{Ar}$$

$$\downarrow$$

$$\frac{N_{He} KT}{V_{He}} = \frac{N_{Ar} KT}{V_{AR}}$$

$$\downarrow$$

But $V_{AR} = V_{TOTAL} - V_{He}$

$$N_{He} \cdot (V_{TOTAL} - V_{He}) = N_{AR} \cdot V_{He}$$

$$\downarrow$$

$$V_{He} = \frac{N_{He} \cdot V_{TOT}}{N_{Ar} + N_{He}}$$

Plug in $N_{He} = 1 \text{ mole} \times \frac{6.02 \times 10^{23}}{\text{mole}} = 6.02 \times 10^{23}$

$N_{Ar} = 1.89 \times 10^{23}$

$V_{TOT} = (0.8\text{ m}) \cdot (4.9 \times 10^{-3}\text{ m}^2) \cdot \pi = 0.012$

$$V_{He} = \frac{(6.02 \times 10^{23})(0.012\text{ m}^3)}{(6.02 \times 10^{23} + 1.89 \times 10^{23})} = 0.009\text{ m}^3$$

So the distance of the piston from the left side, @ equilibrium,

$$h = \frac{V_{He}}{\pi r^2} = \frac{0.009}{(\pi)(0.07)^2} = 0.59\text{ m} = \boxed{59\text{ cm}}$$

To find the change in water temperature, begin with

$$dE = dQ - pdV + \mu dN$$

$$\downarrow \qquad dN = dE = 0$$

$$dQ = pdV$$

So $\Delta Q = \int dQ = \int_{He} pdV + \int_{Ar} pdV$ Plug in $P = \frac{NkT}{V}$

$$= kT\left[N_{He}\int_{He}\frac{dV}{V} + N_{Ar}\int_{Ar}\frac{dV}{V}\right]$$

$$= \left(1.381\times10^{-23}\,\frac{J}{K}\right)(298.15\,K)\left[6.02\times10^{23}\ln\left(\frac{.59}{.30}\right) + 1.89\times10^{23}\ln\left(\frac{.22}{.50}\right)\right]$$

$$\downarrow$$

$$\Delta Q = 1036.7\ J$$

Use $c_p \equiv \left.\frac{dQ}{dT}\right)_p \longrightarrow \Delta T = \frac{1}{c_p}\Delta Q$

$$\downarrow$$

$$\Delta T = \frac{1036.7\ J \times \frac{1\ cal}{4,184\ J} \times 10^3\ g}{1\ cal/g/°} = \boxed{0.25}$$