Physics 301

Homework due 19 October 2022

1) Stowe problem 12.2.

2) Consider an ideal gas, the ratio of whose molar specific heats is given by $\gamma = \frac{c_p}{c_V}$.

The gas is thermally insulated and allowed to expand quasi-statically from an initial volume V_i to a final volume V_f.

a) Use the relation, $pV^\gamma = \text{constant}$, to find the final temperature T_f of the gas.

b) Explain why the entropy does not change during the process.

c) Use the fact that the entropy remains constant to calculate the final temperature T_f of the gas.

3) Liquid mercury at atmospheric pressure and $0°C$ has a molar volume of 14.72 cm^3 / mole and a specific heat at constant pressure of $c_p = 28$ Joules/mole/degree. Its coefficient of volume expansion is $\beta = 1.81 \times 10^{-4}$ / degree, and its compressibility is $\kappa = 3.88 \times 10^{-12}\,\text{cm}^2$/dyne. Find its specific heat c_V at constant volume.

4) Stowe problem 12-3, parts a, b, and d.

5) Stowe problem 12-10.

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Answers to homework due 19 October 2022

① a) Ideal gas: no internal structure, not in an external field, all energy is kinetic. Here, only momentum in one direction is allowed, so the # dof per molecule is 1.

Total # dof $= \boxed{N}$

b) $\boxed{\Omega = (\text{constant}) \cdot L^{N} \cdot E^{N/2}}$

c) $S = k \ln \Omega = k \ln \left[(\text{const}) \cdot L^{N} \cdot E^{N/2} \right]$

$$= \boxed{k \ln(\text{constant}) + N k \ln L + \frac{N}{2} k \ln E}$$

d) $dE = dQ - dW + \underbrace{\mu dN}_{\text{set this} = 0}$

Plug in $dQ = TdS$
and $dW = FdL$

$\boxed{dE = TdS - FdL}$

and, dividing by T,

$$\boxed{dS = \frac{1}{T} dE + \frac{F}{T} dL}$$

e) To get the Ideal One-dimensional Gas Law, begin with

$F_\alpha = T \cdot \frac{dS}{dx_\alpha}$ Here $F_\alpha = F$ and $x_\alpha = L$

$$\frac{dS}{dL} = \frac{d}{dL}\left[k\ln(\text{const}) + Nk\ln L + \frac{Nk}{2}\ln E\right]$$

$$F = \frac{Nk}{L}$$

So $\quad F = T \cdot \frac{Nk}{L}$, or $\boxed{FL = NkT}$

(2) a) Begin with $pV^{\gamma} = \text{constant}$ "Eq 1"

Recall the ideal gas law: $pV = NkT$

$$\downarrow$$

$$\frac{pV}{T} = Nk = \text{constant}' \quad \text{"Eq 2"}$$

Divide Eq 1 by Eq 2:

$$\frac{pV^{\gamma}}{\left(\frac{pV}{T}\right)} = \frac{\text{constant}}{\text{constant}'} = \text{constant}''$$

$$\downarrow$$

$$TV^{\gamma-1} = \text{constant}''$$

This means $T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$, or

$$\boxed{T_f = T_i\left(\frac{V_f}{V_i}\right)^{\gamma-1}}$$

b) When a process is quasi-static, $dQ = TdS$ (Schroeder Eq. 9.14).

This problem concerns a thermally insulated system, so $dQ = 0$

Therefore $dS = 0$.

c) Begin with the First Law: $dE = TdS - pdV + \mu dN$

Use the fact that $dS = 0$ and $dN = 0$.

$$\downarrow$$

$$dE = -pdV \quad \text{"Eq}$$

Also $E = \frac{\nu}{2} NkT$

$\downarrow$

$$dE = \frac{\nu}{2} Nk\,dT \qquad \text{"Eq 4"}$$

Equating the right hand sides of Eq 3 and Eq 4:

$$-p\,dV = \frac{\nu}{2} Nk\,dT \qquad \text{"Eq 5"}$$

The Ideal gas law also relates p, V, N, k, T. Divide Eq 5 by the Ideal Gas Law:

$$\frac{-\cancel{p}\,dV}{\cancel{p}V} = \frac{\nu}{2}\frac{\cancel{Nk}\,dT}{\cancel{Nk}T}$$

$$\downarrow$$

$$-\frac{dV}{V} = \frac{\nu}{2}\frac{dT}{T}$$

$$\downarrow$$

$$(\gamma-1)\frac{dV}{V} + \frac{dT}{T} = 0 \qquad \text{where } \frac{2}{\nu} = \gamma - 1$$

Integrate:

$$(\gamma-1)\ln V + \ln T = \text{constant}$$

Exponentiate: $\boxed{TV^{\gamma-1} = e^{(\text{const})} = \text{const}'}$

So again $\boxed{T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1}}$

(3) $p = 1$ atm.
$T = 0°C$
$v = \frac{V}{\text{mole}} = 14.72$ cm^3/mole

$c_p = 28$ J/mole/degree $= \left.\frac{dQ}{dT}\right)_p$
$\beta = 1.81 \times 10^{-4}$ /degree $= \frac{1}{V}\left.\frac{dV}{dT}\right)_p$
$K = 3.88 \times 10^{-12}$ cm^2/dyne $= -\frac{1}{V}\left.\frac{dV}{dp}\right)_T$

$$c_v \equiv \left.\frac{dQ}{dT}\right)_V$$

Begin with

$$c_v \equiv \left.\frac{dQ}{dT}\right)_V = T\left.\frac{dS}{dT}\right)_V \quad \text{and}$$

$$C_p \equiv \left.\frac{dQ}{dT}\right)_p = T\left.\frac{dS}{dT}\right)_p$$

$$dQ = TdS = T\left[\left.\frac{dS}{dT}\right)_p dT + \left.\frac{dS}{dp}\right)_T dp\right]$$

$$= C_p dT + T\left.\frac{dS}{dp}\right)_T dp$$

So $C_v = \left.\frac{dQ}{dT}\right)_V = C_p + T\left.\frac{dS}{dp}\right)_T \left.\frac{dp}{dT}\right)_V$

From Maxwell's Relation M10 (Stowe p. 217): $\left.\frac{dS}{dp}\right)_T = -\left.\frac{dV}{dT}\right)_p$:

Also $dV = \left.\frac{dV}{dT}\right)_p dT + \left.\frac{dV}{dp}\right)_T dp$, so $\left.\frac{dp}{dT}\right)_V = \frac{-\left.\frac{dV}{dT}\right)_p}{\left.\frac{dV}{dp}\right)_T} = \frac{-\beta V}{-KV}$

So $C_V = C_p + T\left(-\beta V\right)\cdot\left(\frac{\beta}{k}\right)$

$\downarrow$

$$C_V = C_p - \frac{VT\beta^2}{k}$$

Plug in:

$$c_v = \frac{C_V}{\text{mole}} = \frac{C_p}{\text{mole}} - \frac{VT\beta^2}{k\cdot\text{mole}}$$

$$= 28\,\frac{J}{\text{mole}/^\circ} - \left(\frac{14.72\,cm^3}{\text{mole}}\right)\left(\frac{m^3}{10^6\,cm^3}\right)\frac{(273.15^\circ K)(1.81\times 10^{-4})^2}{3.88\times 10^{-12}\,\frac{cm^2}{\text{dyne}}\times\frac{\text{dyne}/cm^2}{10^{-1}\,N/m^2}}$$

$$= \boxed{24.6\ J/\text{mole}/^\circ}$$

(4) $\Omega = (\text{const})\, e^{\alpha V^{4/5}} E^{2N}$ for N particles

(a) $S = k \ln \Omega$

$= k \ln \left[\text{const}\, e^{\alpha V^{4/5}} E^{2N} \right]$

$= k \left[\ln \text{const} + \alpha V^{4/5} + 2N \ln E \right]$

(b) $\frac{1}{T} = \left.\frac{\partial S}{\partial E}\right|_V$

$= \frac{2Nk}{E}$

So $E = 2NkT$

(d) $\frac{P}{T} = \left.\frac{\partial S}{\partial V}\right)_E = k\alpha \cdot \frac{4}{5} V^{-1/5}$

(5) $p = 100$ atm

$v = 0.3$ liter

$a = 5.5 \dfrac{\text{liters}^2\text{-atm}}{\text{mole}^2}$

$b = 0.03 \dfrac{\text{liters}}{\text{mole}}$

Find $c_p - c_v$

Eq 12.17: $c_p - c_v = p \left.\dfrac{dv}{dT}\right)_p$

From Eq. 12.20:

$$c_p - c_v = \frac{R}{1 + \left(\frac{a}{pv^2}\right)\left[\frac{2b}{v} - 1\right]}$$

Use $R = 8.21 \times 10^{-2} \dfrac{\text{liter-atm}}{°K\text{-mole}}$

$$c_p - c_v = \frac{8.21 \times 10^{-2}}{\left\{1 + \left(\frac{5.5}{100 \cdot 0.3^2}\right)\left[\frac{(2)(0.03)}{0.3} - 1\right]\right\}} = 0.16 \text{ J}/°\text{K-mole}$$