Physics 301

Homework due 12 October 2022

1) One kilogram of water at $0^\circ C$ is brought into contact with a heat reservoir at $100^\circ C$. When the water has reached $100^\circ C$,

(a) what has been the change in entropy of the water?

(b) what has been the change in entropy of the reservoir?

(c) what has been the change in entropy of the entire system?

2) Stowe problem 9-18.

3) Stowe problem 9-23.

4) Find the work necessary to inflate a spherical soap bubble to a radius R in the atmosphere. The surface tension of the soap is $\mathfrak{I}$.

5) Consider a system whose equation of state is given by $\frac{p^2}{T^{1/3}}e^{aV} = b$, where p is pressure, T is absolute temperature, V is volume, and a and b are constants. Calculate the

(a) isothermal compressibility

(b) coefficient of volume expansion

Physics 301
Homework due 12 Oct 2022

① $\Delta S = \int dS = \int \frac{dQ}{T}$; T is in Kelvin, so convert: $T_{initial} = 0°C = 273$
$T_{final} = 100°C = 373$

a) Use the fact that $dQ = C\,dT$. For water, $C = 1$ Cal/g/°C

$$\Delta S = Cm\int \frac{dT}{T} = Cm \ln T \Big|_{273.16}^{373.16} = Cm \ln\left(\frac{373.16}{273.16}\right) = Cm \ln 1.37$$

Plug in $m = 1000$ g, and C:

$$\Delta S_{water} = (1000g)(1\ Cal/g/°C) \cdot \ln 1.37 = \boxed{312\ Cal/°C}$$

b) $\Delta S = \int dS = \frac{1}{T}\int dQ$ (T is constant for the reservoir

$\int dQ$ for the reservoir must be the same as $-\Delta Q$ for the kilogram of water, since energy is conserved. For the water,

$$-\Delta Q = -C\int dT = -C(373.16 - 273.16) \cdot 1000g \cdot \frac{1°C}{1°K} = -10^5$$

$$\Delta S_{res} = \frac{1}{373.16\ °K} \cdot (-10^5\ Cal) \times \frac{°K}{°C} = \boxed{-268\ Cal/°C}$$

c) $\Delta S_{system} = \Delta S_{water} + \Delta S_{res} = 312 - 268 = \boxed{44\ Cal/°C}$

2 a) # dof/molecule = 5
molecules = 6.02×10^{23}

$$\#dof_{TOT} = (5)\,6.02\times10^{23} = \boxed{3.01\times10^{24}}$$

b) $E = \frac{1}{2}NkT$

$\downarrow$

$T = \frac{2E}{Nk}$. Plug in $E = 3200\,J$
$N = 3.01\times10^{24}$
$k = 1.381\times10^{-23}\,J/K$

$$\boxed{T = 154\,K}$$

c) $\frac{\sigma}{E} = \sqrt{\frac{2}{N}} = \sqrt{\frac{2}{3.01\times10^{24}}} = 8.15\times10^{-13}$, $\boxed{8.15\times10^{-11}\,\%}$

3.) Begin with $\mathcal{N} = \nu N$.

a) $1 \text{ gram} \times \frac{\text{mole}}{18 \text{ grams}} \times \frac{6.02\times10^{23} \text{ molecules}}{\text{mole}} = \boxed{3.34\times10^{22}}$

b) $\Delta Q = C\Delta T$

$\Delta Q = (1 \text{ Cal/g/}^\circ\text{C}) \times (1\text{K}) \times \left(\frac{1^\circ\text{C}}{1\text{K}}\right) = \boxed{1 \text{ calorie}}$

c) $\Delta E = \frac{1}{2}\mathcal{N}k\Delta T = \frac{1}{2}\nu N k\Delta T$, so

$$\nu = \frac{2\Delta E}{Nk\Delta T} = \frac{2\cdot 1 \text{ Calorie}}{(3.34\times10^{22})\cdot 1.381\times10^{-23}\frac{J}{k}\cdot(1\text{K})\cdot\left(\frac{1 \text{ calorie}}{4.184 J}\right)}$$

$= \boxed{18}$

(4) $W = \int dW$

For surface tension problems, $dW = S \cdot d(\text{Area})$

You might expect then that

$$W(\text{create a sphere of radius } R) = S \times (\text{area of sphere})$$
$$= S \cdot 4\pi R^2$$

However a soap bubble actually has 2 surfaces, inner + outer, so you need to work on both. Thus actually $W = 2 \cdot S \cdot 4\pi R^2$

$$\boxed{W = 8 S \pi R^2}$$

(The student is not strongly penalized for missing the factor of 2.)

5) $\frac{P^2}{T^{1/3}} e^{aV} = b$

a) Find $K \equiv -\frac{1}{V} \frac{\partial V}{\partial p}\Big)_T$

$$e^{aV} = bT^{1/3} p^{-2}$$

$$aV = \ln\left(bT^{1/3} p^{-2}\right)$$

$$V = \frac{1}{a} \ln\left(bT^{1/3} p^{-2}\right)$$

$$= \frac{1}{a} \ln bT^{1/3} - \frac{2}{a} \ln p$$

$$\frac{\partial V}{\partial p}\Big)_T = \frac{-2}{ap}$$

$$K = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = -\frac{a}{\ln\left(bT^{1/3} p^{-2}\right)} \cdot \left(\frac{-2}{ap}\right) = \boxed{\frac{2}{p \ln\left(bT^{1/3} p^{-2}\right)}}$$

b) Find $\beta = \frac{1}{V} \frac{\partial V}{\partial T}\Big)_p$

$$V = \frac{1}{a} \ln\left(bT^{1/3} p^{-2}\right)$$

$$= \frac{1}{a} \ln(bp^{-2}) + \frac{1}{3a} \ln T$$

$$\left.\frac{dV}{dT}\right)_p = \frac{1}{3aT}$$

$$\beta = \frac{1}{V}\left.\frac{dV}{dT}\right)_p = \frac{a}{\ln(bT^{1/3}p^{-2})} \cdot \frac{1}{3aT} = \boxed{\frac{1}{3T\ln(bT^{1/3}p^{-2})}}$$