Physics 301

Homework due 28 September 2022

1) Stowe problem 9-1.

2) Stowe problem 9-5. To do all parts of this problem, you need the following additional information:

$$c_1 = \varepsilon^{-10}$$

$$c_2 = \varepsilon^{-8}$$

where $\varepsilon = 10^{-23}\,J$.

3) Stowe problem 9-10.

4) Stowe problem 9-13.

5) Estimate the probability that a measurement of the air pressure in the classroom will detect that all of the air molecules are in the north half of the room. Discuss numerically how unlikely this is (for example, note how many measurements you would have to make to encounter one instance of this result; compare your calculation to an estimate of the number of rooms available on earth for such a measurement; mention how many years it would take to complete this set of measurements; compare this to the lifetime of the universe, and so forth.)

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Answers to homework due 28 September 2022.

1 a) From Stowe Equation 7.14, $\Omega \propto E^{n/2}$, where $n = \#$ d.o.f.

For an ideal gas $n = \frac{3 \text{ dof}}{\text{molecule}} \times N \text{ molecules} = 3N$.

So for an ideal gas, $\Omega \propto E^{(3N/2)}$

b) $\frac{1}{T} = \frac{dS}{dE}$

$S = k \ln \Omega_0 = k \ln\left[C \cdot E^{3N/2}\right]$

unspecified constant

$\frac{dS}{dE} = k \frac{d}{dE}\left[\ln C E^{3N/2}\right] = k \frac{d}{dE}\left[\ln C + \frac{3N}{2} \ln E\right]$

$= k \cdot \frac{3N}{2} \cdot \frac{1}{E}$

So $\frac{1}{T} = \frac{3kN}{2E} \Rightarrow \boxed{E = \frac{3}{2} NkT}$

(2) a) Since in general $\Omega \propto E^{n/2}$, then

since $\Omega_1 = C_1 E_1^{10}$, n (System A_1) $= 2 \cdot 10 = \boxed{20}$, and

since $\Omega_2 = C_2 E_2^{8}$, n (System A_2) $= 2 \cdot 8 = \boxed{16}$

b) $\Omega_0 = \Omega_1 \cdot \Omega_2 = C_1 C_2 E_1^{10} E_2^{8} = C_1 C_2 (E_1)^{10} (E_0 - E_1)^8$

At equilibrium, $\dfrac{d\Omega_0}{dE_1} = C_1 C_2 \cdot 10 E_1^9 (E_0 - E_1)^8$

$+ C_1 C_2 (E_1)^{10} \cdot 8 \cdot (E_0 - E_1)^7 (-1) = 0$

This means that

$10 E_1^9 (E_0 - E_1)^8 - 8 (E_1)^{10} \cdot (E_0 - E_1)^7 = 0$

Divide through by $2 E_1^9 (E_0 - E_1)^7$:

$5 \cdot 1 \cdot (E_0 - E_1) - 4 \cdot E_1 \cdot 1 = 0$

$\downarrow$

$5E_0 - 5E_1 - 4E_1 = 0$

$5E_0 - 9E_1 = 0$

$E_1 = \dfrac{5E_0}{9} = \dfrac{5 \cdot 10^{-18}\,J}{9} = \boxed{0.556 \times 10^{-18}\,J}$

$E_2 = E_0 - E_1 = 10^{-18}\,J - 0.556 \times 10^{-18}\,J = \boxed{0.444 \times 10^{-18}\,J}$

c) $S = k \ln \Omega_0$

$\Omega_0 = \Omega_1 \cdot \Omega_2 = C_1 C_2 E_1^{10} E_2^{8}$

$= \varepsilon^{-10} \cdot \varepsilon^{-8} \cdot E_1^{10} \cdot E_2^{8}$

Plug in $\varepsilon = 10^{-23}\,J$

$E_1 = 0.536 \times 10^{-18}\,J$

$E_2 = 0.444 \times 10^{-18}\,J$

$$\Omega_0 = (10^{-23})^{-10} \cdot (10^{-23})^{-8} \cdot (0.536 \times 10^{-18})^{10} \cdot (0.444 \times 10^{-18})^{8}$$

$$= 0.00433 \times 10^{90}$$

$$S = k \ln \Omega_0 = \left(1.381 \times 10^{-23} \tfrac{J}{K}\right) \cdot \ln\left[4.33 \times 10^{87}\right] = \boxed{2.79 \times 10^{-21}\,\tfrac{J}{K}}$$

d) $$\frac{1}{T} = \frac{\partial S_1}{\partial E_1} = \frac{\partial}{\partial E_1}\left[k \ln \Omega_1\right] = \frac{\partial}{\partial E_1}\left[k \ln C_1 E_1^{10}\right]$$

$$= k \frac{\partial}{\partial E_1}\left[\ln C_1 + 10 \ln E_1\right]$$

$$= k \cdot 10 \cdot \frac{1}{E_1}$$

So $$T = \frac{E_1}{10\,k} = \frac{0.536 \times 10^{-18}\,J}{(10)\left(1.381 \times 10^{-23} \tfrac{J}{K}\right)} = \boxed{4026\,K}$$

(3) In all cases the change of temperature scale must preserve the value of the energy of the system.

Since energy $E \propto kT$, this means that

$$k_{kelvin} \cdot T_{kelvin} = k_{"R"} \cdot T_{"R"}$$

So $$T_{"R"} = \frac{k_K \cdot T_K}{k_{"R"}}$$

a) $$k_R = 10^{-16} \frac{erg}{°R} \times \frac{10^{-7}\,J}{erg} = 10^{-23} \frac{J}{R}$$

$$T_R = \frac{(1.381 \times 10^{-23}\,J/K) \cdot (373.15\,K)}{(10^{-23}\,J/°R)} = \boxed{515\,°R}$$

b) $k_R = 10^{-4}\ eV/°R$

$$T_R = \frac{(8.63 \times 10^{-5}\ eV/K)\cdot(373.15K)}{(10^{-4}\ eV/°R)} = \boxed{322°R}$$

c) $k = 1\ J/°R$

$$T_R = \frac{(1.381 \times 10^{-23}\ J/K)(373.15\ K)}{(1\ J/°R)} = \boxed{5.15 \times 10^{-21}\ °R}$$

d) $k = 1\ eV/°R$

$$T_R = \frac{(8.63 \times 10^{-5}\ eV/K)\cdot(373.15\ K)}{(1\ eV/°R)} = \boxed{0.0322°R}$$

4) $\Delta S = \frac{\Delta E_1}{T_1} + \frac{\Delta E_2}{T_2}$

Let T_1 = temperature of ice
T_2 = temperature of water
These do not change during the process.

ΔE_1 = change in internal energy of ice.
ΔE_2 = change in internal energy of water.

$$T_1 = -1°C = 272.15K$$

$$T_2 = 10°C = 283.15K$$

$$\Delta E_1 = 1 \text{ calorie}$$

$$\Delta E_2 = -1 \text{ calorie}$$

a) $\Delta S_{ice} = \frac{1 \text{ calorie}}{272.15\ K} = 3.67 \times 10^{-3} \frac{cal}{K}$, or $1.536 \times 10^{-2} \frac{J}{K}$

b) $\Delta S_{water} = \frac{-1 \text{ calorie}}{283.15\ K} = -3.53 \times 10^{-3} \frac{cal}{K}$, or $-1.476 \times 10^{-2} \frac{J}{K}$

c) $\Delta S_{combined} = (3.67 \times 10^{-3}) + (-3.53 \times 10^{-3}) = 1.4 \times 10^{-4} \frac{cal}{K}$

5. $P_{TOT} = \prod_i p(i)$, where the $p(i)$ as the probability that molecule i is in the north half of the room.

Since we are dividing the room in half, $p(i) = \frac{1}{2}$ for all i. Suppose there are 10^{28} molecules, Then

$$P_{TOT} = \left(\frac{1}{2}\right)^{10^{28}}$$