Physics 301

Homework due 7 September 2022

1) Consider the quantity $dF = \left(x^2 - y\right)dx + xdy$.

(a) Explicitly integrate dF over several paths to determine whether or not it is an exact differential.

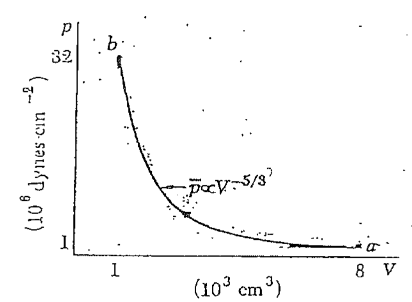
(b) Explicitly differentiate dF to determine whether or not it is an exact differential.

2) Consider a gas whose pressure is proportional to its $(\text{volume})^{-5/3}$.

(a) Use the figure below to determine the constant of proportionality.

(b) Calculate the total work done by the gas in compressing it from state "a" to state "b" (see figure below) in a thermally insulated vessel.

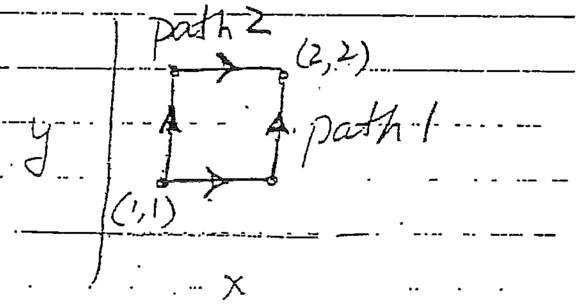
3) Consider a system of 4 particles. Two particles are of a type that can take two states (up or down). These two particles can not simultaneously be in the same state (i.e. both up or both down). The two others are of a type that can take 3 states (A, B, or C). How many states are possible for the combined system?



Physics 301

Answers to homework due 7 Sept 2022

(1) $dF = (x^2 - y)\,dx + x\,dy$



a) Integrate over Path 1:

$$\int_{(1)} dF = \int_{\substack{x_1=1 \\ (y=1,\ \text{fixed})}}^{x_1=2} (x^2-y)\,dx + \int_{\substack{y_1=1 \\ (x=2,\ \text{fixed})}}^{y_2=2} x\,dy = \left(\frac{x^3}{3} - 1\cdot x\right)\Big|_1^2 + 2\cdot y\Big|_1^2$$

$$= \frac{8}{3} - 2 - \frac{1}{3} + 1 + 4 - 2 = 3\tfrac{1}{3}$$

Integrate over Path 2:

$$\int_{②} dF = \int_{y_1=1}^{y_2=2} x\,dy + \int_{x_1=1}^{x_2=2} (x^2-y)\,dx = 1\cdot y\Big|_1^2 + \left(\frac{x^3}{3} - 2x\right)\Big|_1^2$$

$(x=1, \text{fixed})$ $(y=2, \text{fixed})$

$$= 2 - 1 + \frac{8}{3} - 4 - \frac{1}{3} + 2 = \boxed{\frac{4}{3}}$$

As the integrals give different answers, dF is an inexact differential.

b) Let $g = x^2 - y$ and $h = x$

$$\frac{\partial g}{\partial y} = -1$$

$$\frac{dh}{dx} = 1$$

$\frac{\partial g}{\partial y} \neq \frac{dh}{dx}$, confirming that dF is inexact.

2) a) $P = CV^{-5/3}$

When $P = 32$, $V = 1$, so

$$C = \frac{32 \times 10^6 \text{ dynes/cm}^2}{(10^3 \text{ cm}^3)^{-5/3}} = \boxed{3.2 \times 10^{12} \text{ dynes-cm}^3}$$

b) $W = -\int_{V_a}^{V_b} P \cdot dV$

$$= -C \int_{V_a}^{V_b} V^{-5/3} dV$$

$$= -C \left(\frac{-3}{2} V^{-2/3} \right) \Big|_{V_a}^{V_b}$$

$$= \frac{3}{2} C \left(V_b^{-2/3} - V_a^{-2/3} \right)$$

$$= \frac{3}{2} \left(3.2 \times 10^{12} \text{ dynes-cm}^3 \right) \left[\frac{1}{(10^3 \text{ cm}^3)^{2/3}} - \frac{1}{(8 \times 10^3 \text{ cm}^3)^{2/3}} \right]$$

$$= 3.6 \times 10^{10} \text{ dynes-cm}$$

$$= \boxed{3.6 \times 10^{10} \text{ ergs}}$$

3) $3 \times 3 \times 2 = 18$ states

The second 2-state particle does not contribute.