Heisenberg’s Microscope

M. Gold, physics 491

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elements are all necessary for the description of various aspects of the phenomena. From the point of view of the experimenter, the complementarity principle asserts that the physical apparatus available to him has such properties that more precise measurements than those indicated by the uncertainty principle cannot be made.

This is not to be regarded as a deficiency of the experimenter or of his techniques. It is rather a law of nature that, whenever an attempt is made to measure precisely one of the pair of canonical variables, the other is changed by an amount that cannot be too closely calculated without interfering with the primary attempt. This is fundamentally different from the classical situation, in which a measurement also disturbs the system that is under observation, but the amount of the disturbance can be calculated and taken into account. Thus the complementarity principle typifies the fundamental limitations on the classical concept that the behavior of atomic systems can be described independently of the means by which they are observed.

LIMITATIONS ON EXPERIMENT

In the atomic field, we must choose between various experimental arrangements, each designed to measure the two members of a pair of canonical variables with different degrees of precision that are compatible with the uncertainty relations. In particular, there are two extreme arrangements, each of which measures one member of the pair with great precision. According to classical theory, these extreme experimental arrangements complement each other; the results of both may be obtained at once and are necessary to supply a complete classical description of the system. In actuality, however, the extreme complementary experiments are mutually exclusive and cannot be performed together.

It is in this sense that the classical concept of causality disappears in the atomic field. There is causality insofar as the quantum laws that describe the behavior of atoms are perfectly definite; there is not, however, a causal relationship between successive configurations of an atomic system when we attempt to describe these configurations in classical terms.

DISCUSSION OF MEASUREMENT

In this section we consider three fairly typical measurement experiments from the point of view of the new quantum mechanics. The first two are designed to determine the position and momentum of a particle by optical methods; the third is the diffraction experiment in Sec. 2.

LOCALIZATION EXPERIMENT

We consider a particular example of the validity of the uncertainty principle, making use of a position-momentum determination that is typical
of a number of somewhat similar experiments that have been discussed in connection with measurements on particles and radiation fields. We shall consider here the accuracy with which the \( x \) component of the position and momentum vectors of a material particle can be determined at the same time by observing the particle through a rather idealized microscope by means of scattered light.

The best resolving power of the lens \( L \) shown in Fig. 2 is known (either experimentally or from the theory of wave optics) to provide an accuracy

\[
\Delta x \sim \frac{\lambda}{\sin \varepsilon}
\]

in a position determination, where \( \lambda \) is the wavelength of the radiation that enters the lens, and \( \varepsilon \) is the half angle subtended at the particle \( P \) by the lens. For simplicity, we consider the case in which only one of the light quanta \( Q \) is scattered onto the screen \( S \). Because of the finite aperture of the lens, the precise direction in which the photon is scattered into the lens is not known. Then since Eq. (1.2) states that the momentum of the photon after it is scattered is \( h/\lambda \), the uncertainty in the \( x \) component of its momentum is approximately \( (h/\lambda) \sin \varepsilon \).

The \( x \) components of the momenta of the photon and the particle can be accurately known before the scattering takes place, since there is no need then to know the \( x \) components of their positions. Also, if our position measurement refers to the displacement of the particle with respect to the microscope, there is no reason why the total momentum of the system (particle, photon, and microscope) need be altered during the scattering. Then the uncertainty \( \Delta p_x \) in the \( x \) component of the momen-

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2 See footnote 2, page 3.
MOMENTUM DETERMINATION EXPERIMENT

The experiment just discussed assumes that the momentum of the particle is accurately known before the measurement takes place, and then it measures the position. It is found that the measurement not only gives a somewhat inaccurate position determination but also introduces an uncertainty into the momentum.

We now consider a different experiment in which the position is accurately known at the beginning and the momentum is measured. We shall see that the measurement not only gives a somewhat inaccurate momentum determination but also introduces an uncertainty into the position. We assume that the particle is an atom in an excited state, which will give off a photon that has the frequency \( \nu \), if the atom is at rest. Because of the doppler effect, motion of the atom toward the observer with speed \( v \) means that the observed frequency is given approximately by

\[
\frac{\nu}{\nu_0} = 1 + \frac{v}{c}
\]

so that

\[
\nu = \nu_0 \left( 1 + \frac{v}{c} \right)
\]

Accurate measurement of the momentum \( p \) by measurement of the frequency \( \nu \) requires a relatively long time \( \tau \); the minimum error in the
Theoretical Critique: A P Lund and H M Wiseman, New Journal of Physics 12 (2010) was followed by an experiment, summarized for the non-expert Certainty of Uncertainty. Quoting from above reference,

“When first taking quantum mechanics courses, students learn about Heisenberg’s uncertainty principle, which is often presented as a statement about the intrinsic uncertainty that a quantum system must possess. Yet Heisenberg originally formulated his principle in terms of the observer effect: a relationship between the precision of a measurement and the disturbance it creates, as when a photon measures an electron’s position. Although the former version is rigorously proven, the latter is less general and as recently shown mathematically incorrect. In a paper in Physical Review Letters, Lee Rozema and colleagues at the University of Toronto, Canada, experimentally demonstrate that a measurement can in fact violate Heisenberg’s original precision-disturbance relationship.”

The experimental paper: PRL 109, 100404 (2012)

There is controversy, are-weak-values-quantum-dont-bet-it and paper Phys. Rev. Lett. 113, 120404 (2014) (by Christopher Ferrie and Joshua Combes, Center for Quantum Information and Control, University of New Mexico, Albuquerque, New Mexico) claiming that the Rozema result can be obtained classically and is therefore not quantum mechanics but classical statistics (a Monty Hall type paradox). Further theoretical analysis supporting Heisenberg: PRL 111, 160405 (2013)

“We have shown that despite recent claims to the contrary, Heisenberg-type inequalities can be proven that describe a trade-off between the precision of a position measurement and the necessary resulting disturbance of momentum and vice-versa.”

Professor Werner said: “Since I was a student I have been wondering what could be meant by an ‘uncontrollable’ disturbance of momentum in Heisenberg’s Gedanken experiment. In our theorem this is now clear: not only does the momentum change, there is also no way to retrieve it from the post measurement state.”

Professor Lahti added: “It is impressive to witness how the intuitions of the great masters from the very early stage of the development of the then brand new theory turn out to be true.”