Goal of Experiment

- Measure a parameter: statistical precision, accuracy (systematic effects)
- Test a hypothesis: confidence level, goodness of fit

Example: \( c = (3.09 \pm 0.15) \times 10^8 \) m/s, best value (mean) and uncertainty

*Units! No more than two significant digits in the error!*

precision is

\[
\frac{0.15}{3.09} \approx 5\%
\]
"It is well recognized that we can never measure any physical magnitude exactly, that is with zero error.... Therefore, in reporting the result of any measurements, it is obligatory to specify the probability that the result is in error by some specified amount, because a *gamble on relative correctness is always involved in all physical determinations*.

Uncertainties ("errors") are of two types, strictly speaking, should be quoted separately although sometimes they are added in quadrature:

\[
(\delta c)^2 = (\delta_{statistical})^2 + (\delta_{systematic})^2
\]
Statistical and systematic uncertainties

- statistical – accidental or random errors; repeated measurements should be distributed according to normal (Gaussian) about the mean.

- systematic – functions of experimental method, instruments or environmental conditions (e.g. uncertainty in calibrations); effects the accuracy of the measurement; crucial test is reproducibility of result done by different experiments using different methods.
Suppose you make N measurements of a quantity “q”:

\[
\text{mean} \equiv \bar{q} = \frac{1}{N} \sum q_i
\]

\[
\text{variance} \equiv \sigma_q^2 \equiv (\Delta q)^2 = \frac{1}{N-1} \sum (q_i - \bar{q})^2,
\]

Where the statistical error \( \sigma_q \) is the square-root of the variance. Note that the ”−1” results from having determined \( \bar{q} \) from the same data.
If systematic errors are negligible compared to the statistical (measurement is ”statistics dominated”), then the probability content of the statistical error $\sigma_q$ is given by the Gaussian distribution. The probability of observing $\bar{q}$ given that the true value is the theoretical $q_{th}$ is:

| $|\bar{q} - q_{th}|/\sigma_q$ | Probability   \ | Sigma     |
|-----------------------------|--------------|-----------|
| $< 1$                       | 68.27%       | “one-sigma” |
| $< 2$                       | 95.45%       | “two-sigma” |
| $< 3$                       | 99.73%       | “three-sigma” |
| $< 4$                       | $6.3 \times 10^{-3}$ | “four-sigma” |
| $< 5$                       | $5.7 \times 10^{-5}$ | “five-sigma” |
| $< 6$                       | $2.0 \times 10^{-7}$ | “six-sigma” |

So, if the experimental value differs significantly (> three sigma), then either:

- there is some unknown systematic that hasn’t been accounted for;
- theory is wrong!
suppose $q(x^1, x^2, ..., x^n)$, where the $x^i$ are $N$ independent (uncorrelated) quantities ("variables"). An error of $\sigma_{x^i}$ contributes an error of

$$\frac{\partial q}{\partial x^i} \sigma_{x^i}$$

to $\sigma_q$. These are added in quadrature:

$$\sigma_q^2 = \sum_{i=1}^{N} \left( \frac{\partial q}{\partial x^i} \sigma_{x^i} \right)^2 \text{ uncorrelated } x^i$$
To determine if two variables are uncorrelated, make a scatter plot: \( \{ x_j^1, x_j^2 \} \) measurements of \( x^1 \), \( x^2 \).

Figure: Uncorrelated random variables
Correlated Variables

Figure: (Gaussian) Correlated random variables

Start with the simplest type of correlation, a **linear** correlation.