Work done by gas:

Recall \( W = \int F \, dx \) mechanical work.

\[ F = \text{piston force applied to piston, maintaining gas pressure} \]

\[ F = pA \]

Expansion of gas does work:

\[ W = \int pA \, dx = \int p \, dV \]

Process 1 → 2 an example of thermodynamic process leading to change of state.

Here, thermodynamic path is reversible or quasi-equilibrium process.
Joek's Experiment

Adiabatic \((Q = 0)\) free expansion of ideal gas.

Stop cock is opened and gas expands from \(V\) to \(2V\). \(T\) of gas does not change. Example of non-equilibrium, irreversible process.

\[
P_1 \quad 1 \quad 2 \quad \text{Path } 1 \rightarrow 2,
\]

\[\frac{P_1}{2} = P_2 \]

Gas does no work. \(T\) and internal energy \(U\) do not change: \(U(T)\). Internal energy depends only on \(T\).
First Law

\[ \Delta U = Q - W \]

- \( W > 0 \): work done by system
- \( Q > 0 \): heat absorbed by system

\( U \) is a state variable - does not depend on thermodynamic path.

\( Q, W \) are path dependent

**Example:** work done in isothermal, reversible expansion of ideal gas

\[ p = \frac{1}{V} nRT \]

\[ W = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} nRT \frac{dV}{V} = nRT \ln \left( \frac{V_2}{V_1} \right) \]

\[ \Delta U = 0 \implies Q = W \]
(4) molar specific heat of ideal gas

Definition of \( C_p \): \( Q_{ab} = n \ C_p \ (T_1 - T_2) \)

\[ \Delta U_{ab} + \Delta U_{bc} = 0 \]

\[ \Delta U_{ab} = n \ C_p \ (T_1 - T_2) - p \ (V_b - V_a) \]

\[ \Delta U_{bc} = n \ C_v \ (T_2 - T_1) \]

\[ n \ C_p \ (T_1 - T_2) - p \ (V_b - V_a) = n \ C_v \ (T_1 - T_2) \]

From ideal gas law,

\[ p \ (V_b - V_a) = n \ R \ (T_1 - T_2) \]

\[ C_p - R = C_v \]

\[ \boxed{C_p = C_v + R} \]
6. Adiabatic expansion of ideal gas

\[ \Delta Q_{ab} = 0 \]

\[ \Delta U_{ab} = -W_{ab} = \Delta U_{ac} + \Delta U_{cb} \]

(isothermal expansion) \[ \Delta U_{ac} = 0 \]

\[ \Delta U_{cb} = nC_v(T_2 - T_1) \]

\[ W_{ab} = -nC_v(T_2 - T_1) = nC_v(T_1 - T_2) \]

From ideal gas law, \[ nT = pV/k \]

\[ W_{ab} = \frac{C_v}{k} (p_1V_1 - p_2V_2) \]

(adiabatic expansion)

\[ p \frac{dV}{dT} = -nC_v \]

\[ a = p \frac{dV}{dT} \bigg|_{Q=0} = -nC_v \]
From ideal gas law,

\[ p = \frac{nRT}{V} \]

\[ \frac{nRT}{V} \frac{dV}{dT} = -nC_v \]

\[ \frac{dV}{V} = -\frac{C_v}{R} \frac{dT}{T} \]

\[ -\frac{R}{C_v} \ln \frac{V_2}{V_1} = \ln \frac{T_2}{T_1} \]

\[ V_1 \frac{R}{C_v} T_1 = V_2 \frac{R}{C_v} T_2 = \text{Const} \]

\[ V \frac{R}{C_v} T = \text{Const} \]

Adiabatic expansion of ideal gas

or with \( \gamma = \frac{C_v}{C_p} = \frac{R}{C_v} \),

\[ pV^\gamma = \text{Const} \]

\[ pV = \text{Const} \]

\[ 1 + \frac{R}{C_v} = \frac{C_v + R}{C_v} = \frac{C_p}{C_v} = \gamma \quad (\frac{R}{C_v} = \gamma - 1) \]

Compare to isothermal expansion: \( PV = \text{Const} \).

Note, same factor \( \gamma \) come into speed of sound in gas,

\[ v_{\text{sound}} = \sqrt{\frac{\gamma R T}{m}} = \sqrt{\frac{\gamma P}{\rho}} \]
Diesel engine Cycle:

\[ P \rightarrow b \rightarrow c \rightarrow d \rightarrow a \]

- a\to b \text{ adiabatic compression}
- b\to c \text{ fuel ignition, heating of gas}
- c\to d \text{ adiabatic expansion}
- d\to a \text{ cooling of exhaust gas, constant volume}

\[ V_1/V_2 = \text{compression ratio} \]

\[ P_{ab} \]

\[ \text{Net work} = \int P \, dV \]

\[ \eta = \frac{W}{P_{ab}} \]

Theoretical Diesel cycle efficiency \( \approx 0.7 \)