I. Permanent Magnets
Iron has a property of being able to be magnetized.

Magnet has 2 poles: N North seeking S South seeking
when N-S repel, N N's attract via magnetic field \( \mathbf{B}(\mathbf{r}) \).

Field configuration of bar magnet:

\[ \text{magnetic dipole } \mu \quad \text{like} \quad \text{electric dipole } \mathbf{p} = q \mathbf{d} \]

Except no isolated magnetic poles "monopoles" have ever been observed.

Magnetic field lines have no end-points - are always closed loops.

\[ \mathbf{E} = \mathbf{\nabla} \times \mathbf{B} \]
II. Lorentz Force

Force on charged particle depends on particle velocity:

\[ \vec{F} = q \left( \vec{v} \times \vec{B} \right) \]

An example of another velocity dependent force is Stokes law for air resistance:

\[ \vec{F} = -6 \nu \vec{v} \]  \( \vec{v} \) is motion relative to fluid

For Lorentz force, \( \vec{v} \) is relative to inertial observer.

<table>
<thead>
<tr>
<th>Units</th>
<th>Tesla (T)</th>
<th>( 1 \text{T} = \frac{N}{C \text{(m/s)}} = \frac{N}{A \text{m}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss (G)</td>
<td>( 1 \text{G} = 10^{-4} \text{T} )</td>
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Earth's magnetic field \( \sim 10^{-5} \text{T} \)
Typical bar magnet \( \sim 10^{-2} \text{T} \)
Laboratory electromagnet \( \sim 1 \text{T} \)

In presence of \( \vec{E}, \vec{B} \) Lorentz Force

\[ \vec{F} = q \left( \vec{E} + \vec{v} \times \vec{B} \right) \]
$E$ can do work:

$$W_E = \Delta KE = q \int \vec{E} \cdot d\vec{r}$$

$B$ cannot do work:

$$W_B = q \int (\vec{E} \times \vec{B}) \cdot d\vec{r} = 0$$

since $d\vec{r}$ is parallel to $\vec{v} = \frac{d\vec{r}}{dt}$

power delivered $= \vec{F} \cdot \vec{v} = 0$

III. Motioi in uniform $\vec{B}$

first, take $\vec{v} \perp \vec{B}$

uniform circular motion: $m \left( \frac{v^2}{r} \right) = qvB$

$m v = p = q B r$
more generally, there will be a component of \( \vec{V} \parallel \vec{B} \) motion will be a helix.

**IV Magnetic Mirror**

In non-uniform \( \vec{B} \), motion of particle is helix with radius that keeps flux though cube a constant, just like for electric field, \( \Phi_B = \int \vec{B} \cdot d\vec{a} \).

Consider components of \( \vec{V} \parallel \vec{B} \) and \( \perp \vec{B} \). \n
\[
\begin{align*}
V_{\parallel}^2 + V_{\perp}^2 &= V^2 = \text{constant} \\
V_{\parallel}(t) &= V_{\parallel}(0) - V_{\perp}(t) = V_{\parallel}(0) - \frac{B_{\perp}(t)}{B_{\parallel}(0)} \\
\end{align*}
\]

At some point, \( V_{\parallel}(t) = 0 \) => particle turns around. Momentum conservation?

Aurora borealis "northern lights" trapped charged particle magnetically confine species.
V. Mass Spectrometry

\[ E_k = qV_0 \]
\[ m \left( \frac{v^2}{r} \right) = q \nu B \]
\[ m = \frac{1}{2} \left( \frac{q^2 B^2 r^2}{V_0} \right) = \frac{q}{2} \left( \frac{B^2 r^2}{V_0} \right) = \frac{B^2 r^2}{8V_0} \]

VI. Cyclotron

Hardlow D-shaped conductor

Uniform \( B \)

Protons start at \( \pi \) and spiral out

\[ V(t) = V_0 \cos(\omega t) \]

Period \( T = \frac{2\pi r}{v} \)

For proton, \( \frac{e}{m_p} = 9.58 \times 10^7 \text{ rad/s} \)

Cyclotron frequency \( f_p = \left( \frac{15.2 \text{ MHz}}{T} \right) B \)

Radio frequency
VI. Crossed $\vec{E}$, $\vec{B}$ + Hall effect

$$\vec{E} = \frac{e}{\varepsilon_0} (\vec{E} + \vec{v} \times \vec{B})$$
$$= \frac{e}{\varepsilon_0} (\vec{E} - \vec{v} \times \vec{B})$$
$$F = 0 \text{ with } \nu = \frac{E}{B}$$

Metallic strip in magnetic field $\vec{B}$ and current $I$

At equilibrium, $F = 0$

Strip dimensions $w (w \times d)$

$$\vec{J} = ne \vec{V}_d$$
$$I = ne V_d (ld)$$

$V_d = \frac{V_{H \times}}{d \times B} = \frac{V_{H \times}}{B}$

$$I = ne \left( \frac{V_{H \times}}{d \times B} \right) ld = ne \left( \frac{V_{H \times}}{B} \right) l$$

Allowing measurement of $n = \left( \frac{I}{e} \right) \frac{B}{V_{H \times} l}$
III. For a on wire

\[ \vec{J} = e n \vec{V}_d \]

\[ I = enV_d a \]

\[ \text{# electrons} = n \cdot a \cdot \Delta l \]

Force on single electron \( \vec{F} = -e \vec{V}_d \times \vec{B} \)

Force on section of wire

\[ \Delta \vec{F} = (\text{# electrons}) \vec{J} = n \cdot a \cdot \Delta l (-e \vec{V}_d) \times \vec{B} \]

Define direction of \( \Delta \vec{l} \) as direction of current \((-eV_d)\)

\[ \Delta \vec{F} = I \Delta \vec{l} \times \vec{B} \]

In limit, \( \Delta \vec{F} \approx I d\vec{l} \times \vec{B} \)
Zero torque in x-y plane

Rotate loop about x-axis:

\[ \vec{a} = (\text{loop area}) \times \vec{z} \]

\[ \vec{r} = \vec{r} - (\vec{z} \times \vec{B}) \]

\[ \gamma = 2 \left( \frac{d}{2} \right) R EB \sin \theta \]

\[ \vec{a} = I \vec{a} \times \vec{B} \]

\[ U(\theta) = \mu B (1 - \cos \theta) \]

\[ U(0) = 0 \]

\[ U(\pi) = 2 \mu B \]
Solenometric: coil & fixed magnet

radial $B^2$ always lies in plane of coil

Sprung coil wound around Fe core

$\ell = NIA = K_{spr}

N$ loops in coil
magnetic Bottle (magnetic confinement)

\[ | \mathbf{v}^2 | = \text{constant} = v_0^2 \]

\[ v_{11}^2 + v_1^2 = v_0^2 \]

\[ v_{11}^2 = v_0^2 - v_1^2 = v_0^2 - v_{10}^2 \left( \frac{3 \varepsilon^2}{B^2} \right) \]

\[ \frac{v_1^2}{B} = \text{constant} \]

\[ H_{60} = 80.84 \]
\[ 72.62 \]
\[ 4.948, 40, 31, 24 \]

\[ m \left( \frac{v_r^2}{r} \right) = 8 \pi B \]

\[ (m \varepsilon r) = 8 \pi r^2 \]

\[ m v_1 = 8 \pi B r \]

\[ m v_{1r} (2 \pi r) = 8 \pi B r \]

\[ m v_{1z} \]

\[ \frac{m v_2}{B} = r \]
force on wire $\vec{F} \propto \frac{\partial \vec{A}}{\partial x}$, say given by $I$.

$\vec{F} = e \vec{v} \times \vec{A}$

$d\vec{F} = en \vec{v} \times \vec{B} \cdot (\vec{v} \cdot d\vec{l})$

$= I \vec{v} \times \vec{B}$

$F = ILB$

$F = IL$