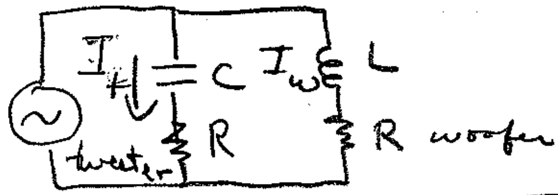
161.002

Spring 2008
M. Gold

Hw #15 Solutions

31.40



a) $Z_T = R + \frac{1}{i\omega C}$; $|Z_T| = \sqrt{R^2 + (1/\omega C)^2}$ tweeter

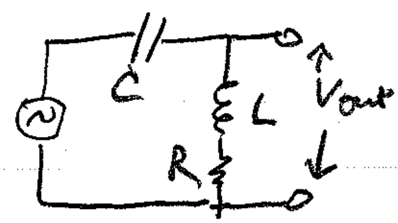
b) $Z_w = R + i\omega L$; $|Z_w| = \sqrt{R^2 + (\omega L)^2}$ woofer

c) if $Z_T = Z_w$, $I_T = I_w$

d) at the crossover point $(Z_T = Z_w)$

$$\omega = \frac{1}{\sqrt{LC}} \; ; \; f = \frac{1}{2\pi\sqrt{LC}}$$

31.49 High-pass filter



$$\frac{V_{out}}{V_{in}} = \frac{R + i\omega L}{\frac{1}{i\omega C} + R + i\omega L} = g e^{i\phi}$$

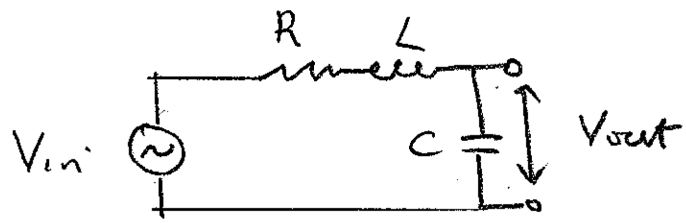
$$|g(\omega)|^2 = \frac{(R + i\omega L)}{R + i(\omega L - \frac{1}{\omega C})} \left(\frac{R - i\omega L}{R - i(\omega L - \frac{1}{\omega C})} \right)$$

$$g(\omega) = \sqrt{\frac{R^2 + (\omega L)^2}{R^2 + (\omega L - \frac{1}{\omega C})^2}} \xrightarrow[\omega \to 0]{} \omega C$$

$$g(\omega) \xrightarrow[\omega \to \infty]{} \frac{\omega L}{\omega L} = 1$$

HW 15

31.50



$$\frac{V_{out}}{V_{in}} = \frac{1/i\omega C}{R+i(\omega L - 1/\omega C)} = g e^{i\phi}$$

$$g(\omega) = \frac{1/\omega C}{\sqrt{R^2+(\omega L-1/\omega C)^2}} \xrightarrow[\omega\to 0]{} \frac{1/\omega C}{1/\omega C} = 1$$

$$g(\omega) \xrightarrow[\omega\to\infty]{} \left(\frac{1}{\omega C}\right)\left(\frac{1}{\omega L}\right) \to 0$$

31.53 In L-R-C circuit, $I(t) = I(\omega)e^{i\omega t}$

$$I(\omega) = \frac{\mathcal{E}_0}{R+i(\omega L-1/\omega C)} = |I(\omega)|\, e^{i\phi}$$

Voltage across capacitor is;

$$V_C(\omega) = \frac{1}{i\omega C} I(\omega) = |V_C(\omega)|\, e^{i\phi_C}$$

$$V_C(t) = |V_C(\omega)| \cos(\omega t + \phi_C)$$
$$I(t) = |I(\omega)| \cos(\omega t + \phi)$$

since $\langle \cos(\omega t+\phi)\rangle_T = \frac{1}{2}$

$$U_E = \frac{1}{4} C |V_C(\omega)|^2 \;;\; U_B = \frac{1}{4} L |I(\omega)|^2$$

Calculating the modulus of $I(\omega)$:

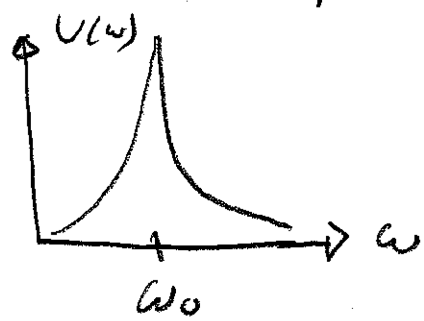
$$|I(\omega)| = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

and $\left|V_C(\omega)\right| = \left|\frac{1}{i\omega C} I(\omega)\right| = \frac{1}{\omega C}\left|I(\omega)\right|$

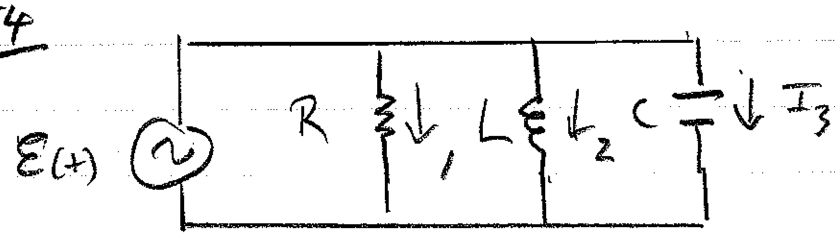
So $U_E = \frac{1}{4}\frac{1}{\omega^2 C}\left|I(\omega)\right|^2$

$U_B = \frac{L}{4}\left|I(\omega)\right|^2$

Both $U_E(\omega)$, $U_B(\omega)$ go to zero at $\omega = 0$ and at $\omega \to \infty$. They both peak at $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$



31.54



$I_1 = \mathcal{E}_0/R$

$I_2 = \mathcal{E}_0/i\omega L$

$I_3 = \mathcal{E}_0\, i\omega C$

$I = I_1 + I_2 + I_3$

$= \mathcal{E}_0\left(\frac{1}{R} + \frac{1}{i\omega L} + i\omega C\right)$

$$|Z|^{-1} = \left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)\right]^{-1/2}$$

Magnitude of current is,

$$|I(\omega)| = \mathcal{E}_0 \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

$$= \sqrt{|I_1|^2 + (|I_2| - |I_3|)^2}$$

31.64 a) $\omega_0 = \left.\frac{1}{\sqrt{LC}}\right|_{\substack{L = 1.8\,H \\ C = 0.9\,\mu F}} = 786 \text{ rad/s}$

b) $|Z(\omega_0)|^2 = \sqrt{R^2 + (\omega_0 L - 1/\omega_0 C)^2} = R = 300\,\Omega$

$$I_{rms}(\omega) = \left.\frac{V_{rms}}{Z(\omega)}\right|_{\omega = \omega_0} = 0.200\,A$$

c) $I_{rms}(\omega) = \frac{1}{2}\frac{V_{rms}}{|Z(\omega_0)|} \Rightarrow \frac{1}{|Z(\omega)|} = \frac{1}{2R}$

$$|Z(\omega)|^2 = 4R^2 = R^2 + (\omega L - 1/\omega C)^2$$

$$(\omega^2 LC - 1)^2 = 3\omega^2 R^2 C^2$$

$$\omega^4 (LC)^2 - \omega^2 (3R^2C^2 - 2LC) + 1 = 0$$

$$\omega^4 - \omega^2 \frac{3R^2C^2 - 2LC}{(LC)^2} + \left(\frac{1}{LC}\right)^2 = 0$$

HW #15

solving quadratic and putting in the values

$$\omega_{\pm}^2 = \begin{cases} 8.90 \times 10^5 \,[\text{rad/s}]^2 \\ 4.28 \times 10^5 \,[\text{rad/s}]^2 \end{cases}$$

$\omega_+ = 943$ rad/s; $\omega_- = 654$ rad/s

(d)

R	$\lvert I(\omega_0)\rvert$	$\omega_+ - \omega_-$
300 Ω	0.200 A	289 rad/s
30 Ω	2 A	28.9 rad/s
3 Ω	20 A	2.89 rad/s