161.002

Spring 2008
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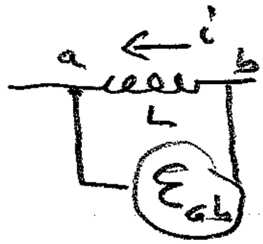
Hw # 14 Solutions

30.10



$\mathcal{E}_{ab} = +1.04\,V$

$\mathcal{E}_{ab} = -L\frac{dI}{dt}$, therefore $\frac{dI}{dt} < 0$ current decreasing

$$\left|\frac{\Delta I}{\Delta t}\right| = \frac{|\mathcal{E}_{ab}|}{L} = \frac{1.04\,V}{0.260\,H} = 4\,A/s$$

$$I(2s) = 12\,A - (2\,s)(4\,A/s) = 4\,A$$

30.11 for uniform B, $\Phi_B = B\,a$

$$B = \left(\mu_0 \frac{N}{\ell} I\right),\quad L = \frac{N\dot{\Phi}_B}{\dot{I}} = \frac{\mu_0 N^2 a}{\ell}$$

30.20 $I(t) = I_0\, e^{-tR/L}$

$$L = \frac{-tR}{\ln(I(t)/I_0)} = \frac{-2\,ms\,(15\,\Omega)}{\ln\left(\frac{.210\,A}{.420\,A}\right)} = 43.3\,mH$$

30.32 $\omega = 2\pi f = \frac{1}{\sqrt{LC}}$

$$C = \frac{1}{4\pi^2 f_{min}^2}\,\frac{1}{L} = \frac{1}{4\pi^2(5.4\times10^{-5}\,Hz)^2(2.37\times10^{-3}\,H)}$$

$$= 36.7\,pF \quad (1\,pF = 10^{-12}\,F)$$

30.40 a) $q(t) = A e^{-\frac{R}{2L}t} \cos(\omega' t + \phi)$

$$\dot{q} = -A\frac{R}{2L} e^{-\left(\frac{R}{2L}\right)t} \cos(\omega' t + \phi) - \omega' A e^{-\frac{R}{2L}t} \sin(\omega' t + \phi)$$

$$\ddot{q} = A\left(\frac{R}{2L}\right)^2 e^{-\frac{R}{2L}t} \cos(\omega' t + \phi) + 2\omega' A\left(\frac{R}{2L}\right) \sin(\omega' t + \phi) - \omega'^2 A e^{-\frac{R}{2L}t} \cos(\omega' t + \phi)$$

$$\ddot{q} + \frac{R}{L}\dot{q} + \frac{q}{LC} = q\left[\left(\frac{R}{2L}\right)^2 - \omega'^2 - \frac{R^2}{2L^2} + \frac{1}{LC}\right] = 0$$

so $\omega' = \left[\frac{1}{LC} - \frac{R^2}{4L^2}\right]^{1/2}$

b) At $t=0$, $q = Q$ & $I = -\dot{q} = 0$

so $A\cos\phi = Q$ and $\dot{q}(0) = -\frac{R}{2L} A\cos\phi - \omega' \sin\phi = 0$

giving $A = \frac{Q}{\cos\phi}$; $\tan\phi = -\frac{R}{2L\omega'}$

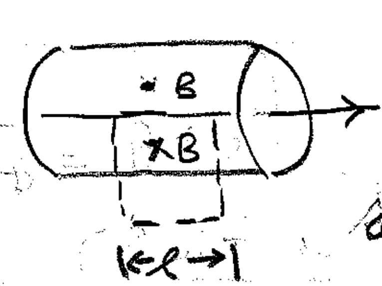
30.48 (a)

Amperian loop

inner conductor radius a
I out of page on inner
I into page on outer

(a) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B(r) = \begin{cases} \frac{\mu_0 I}{2\pi r} & r < B \\ 0 & r > B \end{cases}$

(b)



$da = l\,dr$

$$d\Phi_B = B(r)\,da = \frac{\mu_0 I l}{2\pi} \frac{dr}{r}$$

(c) Integrating,

$$\Phi_B = \int_a^b \frac{\mu_0 I l}{2\pi}\frac{dr}{r} = \frac{\mu_0 I l}{2\pi}\ln\left(\frac{b}{a}\right)$$

(d) $$L = \frac{\Phi_B}{I} = l\,\frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$$

(e) $$U_B = \frac{1}{2}LI^2 = \frac{lI^2\mu_0}{4\pi}\ln(b/a)$$

30.57 $V_C = 12\,V$; $U_C = \frac{1}{2}CV_C^2$

$$C = 2U_C/V_C^2 = 2(0.0160\,J)/(12\,V)^2 = 222\,\mu F$$

$$f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow L = \frac{1}{(2\pi f)^2 C}\Bigg|_{f=3500\,Hz} = 9.31\,\mu F$$

30.59 magnetic field energy density $u_B = B^2/2\mu_0$

$$u_B = 6.366\times10^4\,J/m^3$$

total magnetic energy in volume V, $U_B = Vu_B$.

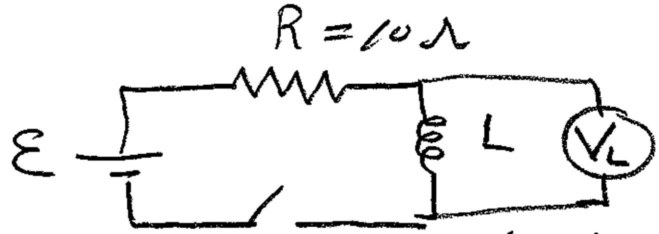
mass of material in sunspot $m = \rho V$, Setting $K = U_B$:

$$\frac{1}{2}(\rho V)v^2 = Vu_B \Rightarrow v = \sqrt{\frac{2u_B}{\rho}} = 2\times10^4\,m/s$$

about $\frac{1}{30}$ the escape velocity from the surface of the sun.

Hw 14

30-61



switch closed @ $t=0$

a) Voltage across R increases with time, whereas Voltage across L decreases with time as in the graph

b) Since $V_L(t) \xrightarrow[t\to\infty]{} \text{constant} \approx 15V$, inductor has some internal resistance (R_L).

c) At $t=0$, $I(0)=0$ so $\mathcal{E} = V_L(t) = 50V$ from graph

d) From graph, $V_L(t) \xrightarrow[t\to\infty]{} I_{max} R_L = 15V$

$$\mathcal{E} - I_{max}(R+R_L) = 0 \;;\; I_{max} = \frac{50V - 15V}{10\,\Omega} = 3.5A$$

e) $R_L = \dfrac{15V}{3.5A} = 4.3\,\Omega$; $R_{TOT} \equiv R_L + R$

$\tau \equiv L/R_{Tot}$

We can get L from τ.

$$I(t) = \frac{\mathcal{E}}{R_{TOT}}\left(1 - e^{-t/\tau}\right)$$

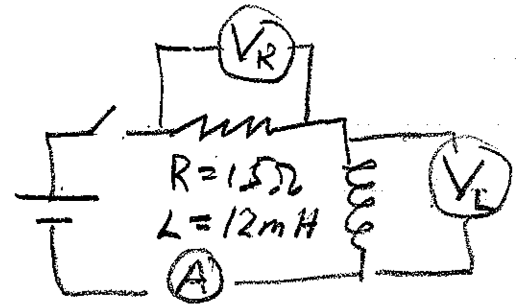
$$V_L(t) = \mathcal{E} - IR = \mathcal{E}\left[1 - \frac{R}{R_{Tot}}\left(1 - e^{-t/\tau}\right)\right]$$

When $t=\tau$, $V_L(\tau) = 50V\left[1 - \frac{10}{14.3}\left(1 - \frac{1}{e}\right)\right] = 27.9V$

From the graph, $\tau \cong 3\ ms$, $L = 14.3\,\Omega(3ms) = \underline{43\ mH}$

HW 14

30-64.



$25V = \mathcal{E}$

switch closed @ $t=0$.
L has negligible internal resistance

a) At $t=0$, $I=0$, $V_R=0$, $V_L = \mathcal{E} = 25V$

b) For $t \gg L/R$ $I = I_{max} = \frac{\mathcal{E}}{R} = 1.67A$
$V_R = 25V$, $V_L = 0$

c) None of these answers depend on L.