28.4

\[ \sin \theta = \sin (180^\circ - \theta) \]

\[ B = \frac{\mu_0 eV}{4\pi r^2} \times \sin 40^\circ \]

\[ = \left(10^{-7} \frac{T \cdot m}{A}\right) \left(1.6 \times 10^{-19} C\right) \left(2.50 \times 10^{-5} m\right) \frac{3(6.43)}{(1.75 \times 10^{-5} m)^2} \]

\[ = \frac{(1.6)(2.5)3(0.643) \times 10^{-3} T}{(1.75)^2} = 2.5 \text{ mT} \]

28.20

Directly below the transmission line, the direction of \( B \) is east.

Down into page

Current into the page

\[ B = \frac{\mu_0 I}{2\pi r} = 2 \times 10^{-7} \frac{T \cdot m}{A} \frac{800A}{5.50 m} = 2.91 \times 10^{-5} T \]

This is nearly equal to the Earth's magnetic field, so the current is really a problem and the hiker should move away from the line to use their compass to determine north.
At $p$, the straight part of $i$ do not contribute.

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{i \, d\vec{r} \times \vec{r}}{r^3} = -\hat{z} \left( \frac{\mu_0 i}{4\pi R^2} \right) \int_0^\Pi R \, d\phi = -\hat{z} \left( \frac{\mu_0 i}{4R} \right)$$

If the loop were $\rightarrow$, $\vec{B}$ would be in the $+\hat{z}$ direction.

28.32 At the center of the coil,

$$B(\phi = 0) = \frac{\mu_0 N I}{2a} \Rightarrow I = \frac{2a B}{\mu_0 N} = \frac{2(0.024m)(1.055T)}{4\pi \times 8 \times 10^{-7} \text{ Tm} \over \text{A}} = 27.77 \text{ A}$$

Along $x$, $B(x) = B(0) \left( \frac{a^3}{x^2 + a^2} \right)^{3/2}$

Solving $\frac{a^3}{(x^2 + a^2)^{3/2}} = 1$

$$(x^2 + a^2)^{3/2} = a^6$$

$x^2 + a^2 = a^{16}$  \Rightarrow  $x = a \left( \frac{a^3}{2} \right)^{1/2}$

$x = 0.766 \, a = (0.024 \, m) \times 0.766 = 0.0184 \, m$
\[ i' = 5 \text{A} \]
\[ \omega = y_2 - y_1 = 7.4 \text{cm} \]
\[ y_2 = 10 \text{cm}; \quad y_1 = 2.6 \text{cm} \]
\[ L = 20 \text{cm} \]

\[ \overrightarrow{B} = -\frac{1}{\lambda} \left( \frac{\mu_0 i'}{2\pi} \right) \frac{1}{y} \]

Force on sides with opposite and equal (due to give a net torque)

\[ \overrightarrow{F} = i' \int d\vec{l} \times \overrightarrow{B} = +\frac{q}{2} L i' \left( B(y_1) - B(y_2) \right) \]

\[ = +\frac{q}{2} \frac{\mu_0 i'}{2\pi} L \left( \frac{1}{y_2} - \frac{1}{y_1} \right) = \]

\[ = +\frac{q}{2} \left( 2 \times 10^{-7} \text{T.m} \right) (5 \text{A})(14 \text{A})(0.2 \text{m}) \left( \frac{1}{0.10 \text{m}} - \frac{1}{0.026 \text{m}} \right) \]

\[ = +7.57 \times 10^{-5} \text{N} \quad (\text{attraction}) \]

\[ B = B_x \]

Field due to coil at center
\[ B_0 = \frac{\mu_0 NI}{2a} \]

Between coils,
\[ B(x) = B_0 \left[ \left( \frac{a^3}{(x - \frac{a}{2})^2 + a^2} \right)^{3/2} + \left( \frac{a^3}{(x - \frac{a}{2})^2 + a^2} \right)^{3/2} \right] \]

\[ = B_0 \left[ \left( \frac{a^3}{(x + \frac{a}{2})^2 + a^2} \right)^{3/2} + \left( \frac{a^3}{(x - \frac{a}{2})^2 + a^2} \right)^{3/2} \right] \]
Midway between coils \( x = 0 \)

\[
B(0) = B_0 \left( \frac{4}{5} \right)^{3/2} = 1.43 B_0
\]

For \( N = 300, \ c = 6A, \ a = 0.08m \)

\[
B_0 = \frac{2 \pi \times 10^{-7} \ T \ m}{\frac{6A}{0.08m}} = 14.14 \ mT
\]

\[
B(0) = 20.2 \ mT
\]

Let \( y = \frac{x}{a} \) \( f(y) = \left[ \frac{(y+\frac{1}{2})^2}{(y+\frac{1}{2})^2+1} \right]^{3/2} + \left[ \frac{(y-\frac{1}{2})^2}{(y-\frac{1}{2})^2+1} \right]^{3/2} \)

\[
\frac{dB}{dx} = \frac{B_0}{a} \frac{df}{dy} \quad ; \quad \frac{d^2B}{dx^2} = \frac{B_0}{a^2} \frac{d^2f}{dy^2}
\]

\[
\frac{dy}{dy} = \left. \left[ \frac{(y+\frac{1}{2})}{(y+\frac{1}{2})^2+1} \right]^{3/2} + \left[ \frac{(y-\frac{1}{2})}{(y-\frac{1}{2})^2+1} \right]^{3/2} \right|_{y=0} = 0
\]

\[
\frac{d^2f}{dy^2} = -3 \left[ \frac{1}{(y+\frac{1}{2})^{5/2}} + \frac{1}{(y-\frac{1}{2})^{5/2}} \right]
\]

\[
-3 \left( -\frac{1}{2} \right)^2 \left[ \frac{(y+\frac{1}{2})^2}{(y+\frac{1}{2})^2+1} \right]^{7/2} + \frac{(y-\frac{1}{2})^2}{(y-\frac{1}{2})^2+1}^{7/2}
\]

\[
\left. \frac{d^2f}{dy^2} \right|_{y=0} = -3 \cdot 2 \left( \frac{4}{3} \right)^{5/2} + \frac{15}{4} \cdot 2 \left( \frac{4}{3} \right)^{5/2} \cdot \frac{4}{5} = 0
\]

Both first and second derivatives vanish at \( x = 0 \) so \( B(x) \) is changing very slowly.
\[ B(y) = B_0 \, \tilde{g}(y) \]

and \[ B_1(y) = B_0 \, \tilde{g}(y), \quad \tilde{g}(y) = \left( y + \frac{1}{2} \right)^{\frac{3}{2}} \]
Field due to single infinite sheet:
\[ \oint \mathbf{B} \cdot d\mathbf{A} = \frac{\mu_0 (N \lambda)}{2} \, dx \]

\[ \mathbf{B} \cdot d\mathbf{A} = \frac{\mu_0 (N \lambda)}{2} \, dx \]

\[ \mathbf{B} = \frac{\mu_0 (N \lambda)}{2} \]

At P, S the vector sum due to sheets cancels. Between the current sheets, the fields add giving
\[ \mathbf{B}_R = x \, \mu_0 \left( \frac{N \lambda}{2} \right) \]