I. Momentum $\vec{p} = m \vec{v}$

Conserved for isolated system

$F_{12} \leftarrow [m_1, m_2] \rightarrow F_{21}$

Spring pushes masses apart

$F_{12} \cdot \Delta t = F_{21} \Delta t$ impulse

$\Delta p_1 = \Delta p_2$ momentum

Momentum is conserved

Example: inelastic $\text{[m]} \rightarrow \text{[5m]}$ collision

$V_f = \frac{mv}{6m}$
II. Kinetic Energy \( (T) \)

\[
T = \frac{1}{2} m v^2
\]

Work \( W = \int F_{\text{net}} \cdot d \)

\( P \) Force \( \Delta \) displacement

Work results in change in K.E.

\[
W = \Delta T
\]

Example: 2 blocks starting from rest

\[
m_1 \quad F_{\text{net}} \quad T_1 = \frac{1}{2} (m_1) v_1^2
\]

\[
m_2 = 4m_1 \quad F_{\text{net}} \quad T_2 = \frac{1}{2} (4m_1) v_2^2
\]

work \( T_1 = T_2 \Rightarrow v_2 = \frac{v_1}{2} \)

\[
\Delta T_1 = \frac{d}{v_1} \quad \Delta T_2 = \frac{d}{v_2} = 2 \Delta T_1
\]

Impulse \( \Delta P_1 = F_{\text{net}} \Delta t_1 \quad \Delta P_2 = F_{\text{net}} \Delta t_2 = 2 \Delta P_1 \)

\[
m_2 v_2 = (4m_1) \frac{v_1}{2} = 2 (m_1 v_1) \quad \checkmark
\]
potential energy (U)

For conservative forces, conservation of energy can be extended.

Friction is a non-conservative force because mechanical energy is lost as heat.

Gravity is a conservative force

![Diagram of a ball dropped from rest at height ho.](image)

\[\text{Work done by gravity} = mgh_0 = \frac{1}{2}mv_f^2\]

\[E = mggh + \frac{1}{2}mv^2 \quad \text{constant}\]

Therefore, define \(U = mgh\) and gravity conserve mechanical energy:

\[E = \frac{1}{2}mv_f^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2\]
example

\[ \text{free body diagram} \]

\[ F_{\text{net}} = W \cdot \frac{h}{g} = mg \cdot \frac{h}{g} \]

Work done by gravity: \[ \frac{1}{2} m g h \]

K.E. of block at bottom: \[ \frac{1}{2} m v^2 = m g h \]

**Example: Simple Pendulum**

\[ \text{Force not constant but we still have} \]

Initial height \[ h_0 \rightarrow v_0 \]

\[ E = \frac{1}{2} m v^2 + m g h \]

\[ v_0 \text{ is then } \frac{1}{2} m v_0^2 = m g h_0 \rightarrow v_0 = \sqrt{2 g h_0} \]

\[ v_0 \text{ does not depend on the mass and only}\]
\[ \text{does the period.} \]

As long as displacement (\( \theta \)) is small, pendulum is "simple" and

\[ \text{period } T = 2 \pi \sqrt{\frac{L}{g}} \]
pictorial representation of conservation of mechanical energy:

\[ E = T + U \]

\[ x_{\text{max}} \quad \text{displacement} \]

IV. Angular momentum

\[ \omega = \frac{\Delta \theta}{\Delta t} \]

\[ \Delta s = r \Delta \theta \quad \text{in radians} \]

\[ v = \frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} = \omega \]

for uniform circular motion, \( v = \frac{2\pi r}{T} \)

So \[ \omega = \frac{2\pi}{T} = 2\pi f \quad f = \text{frequency} \]

\[ T = \text{period} \]

\[ L = mvr \quad \text{conserved for isolated system} \]
For mass on string,

\[ L = mr^2 \omega \]

\( \omega \) = moment of inertia about axis of rotation

String, puck sliding on horizontal frictionless surface with hub

\[ \sqrt{L} \]

\[ (\frac{r}{L})^2 \]

Shortening \( r \):

\[ L = mr_1^2 \omega_1 = mr_2^2 \omega_2 \] (Conserved)

\[ \omega_2 = \left( \frac{r_2}{L} \right)^2 \omega_1 \]

Rotational K.E. increases:

\[ T = \frac{1}{2} mv^2 = \frac{1}{2} mr^2 \omega^2 = \frac{1}{2} I \omega^2 \]

Shortening string does work against centripetal force.

\[ L = I \omega \]

\[ T = \frac{1}{2} I \omega^2 \] true in general

for rigid bodies, where rigid means all parts rotate with same \( \omega \).
Examples of Moments of Inertia

1. Hoop of mass $m$ radius $R$

$$I = mR^2$$

2. Uniform rod of length $l$

$$I = \frac{1}{12} ml^2$$

3. Uniform sphere about central axis

$$I = \frac{2}{5} mR^2$$

V. Understanding Kepler's 2nd Law

"equal areas in equal time"

$$\frac{dA}{dt} = \frac{1}{2} \left( r \frac{d\theta}{dt} \right)$$

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} \frac{mr^2w}{m} = \frac{L}{2m}$$