COPERNICUS "On the Revolutions of the Heavenly Spheres" 1543

Keppler "A New Astronomy" 1609

Newton "Principia" 1687
"the mathematical principles of natural philosophy"

I. Example: retrograde motion of Mars

II. Uniform circular motion

\[ T = \frac{2\pi a}{v} \]
\[ a = \frac{v^2}{r} \]

\[ x^2 + y^2 = r_e^2 \]
\[ \frac{r_e}{c} = \frac{150 \times 10^9 \text{m}}{3 \times 10^8 \text{m/s}} = 500s = \frac{8}{3} \text{ light-minutes} \]
III. Kepler's Laws

1. Planetary orbits are ellipses

\[ a = \text{semi-major axis} \]
\[ b = \text{semi-minor axis} \]
\[ \varepsilon = \text{eccentricity} \]

\[ \varepsilon = 0 \] for circle

| Planet | \( \varepsilon \) | \( a \times 10^{10} \) | \( T \) (y) | \( \frac{T^2}{a^3} \) |
|--------|-----------------|-----------------|--------------|-----------------
| Mercury| 0.2             | 0.179           | 0.241        | 1.3            |
| Venus  | 0.007           | 10.8            | 0.615        |                |
| Earth  | 0.02            | 15.0            | 1.00         |                |
| Mars   | 0.10            | 22.8            | 1.88         |                |
| Jupiter| 0.09            | 77.8            | 11.9         |                |
| Saturn | 0.06            | 143             | 29.5         |                |
| Uranus | 0.05            | 290             | 84.0         |                |
| Neptune| 0.01            | 450             | 165          |                |

Kepler's 3rd Law

\[ T^2 \propto a^3 = \text{constant} \]
#3 Equal areas in equal time - Kepler's 2nd Law

\[ \text{Equal area} \]

\[ \text{at Earth} \]

\[ \text{at} \]

IV. Newton's Universal Law of Gravitation:

\[ F = G \frac{m_1 m_2}{r^2} \]

\[ \text{attractively} \]

\[ r^2 \]

\[ G, \text{ newton's constant} \]

\[ \approx 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 \text{ kg}^{-2} \]

\[ \text{very, very small} \]

"Actua at a distance"
1. Inverse square law

* #2/ elliptical, closed orbits

\[ \frac{\mu V^2}{r} = \frac{\mu GM\omega}{r^2} \]

\[ \omega = \frac{2\pi r}{T} \]

\[ \left( \frac{2\pi r}{T} \right)^2 \frac{1}{r} = \frac{GM\omega}{r^2} \]

\[ T^2 / r^3 = 4\pi^2 / (GM\omega) \]

* when universal law combined with Newton's laws of motion.

precession of perihelion unless force goes exactly as \( 1/r^2 \).
#3 Shell Theorem

\[ F = \frac{m \cdot M_{\text{shell}} G}{r^2} \]

shell of uniform mass density

force between spherically symmetric masses \( \propto \frac{1}{r^2} \)

distance between centers

#4 Ultimately, \( \frac{1}{r^2} \) law follows from dimensionality of space

\[ \text{surface area of sphere} = 4\pi r^2 \]

Does space have tiny hidden dimensions?
IV. Kepler's 2nd Law

Do not depend on force of gravity, but rather follow generally from Newton's laws as

\[ \frac{\Delta \text{Area}}{\text{cent} \text{time}} = \frac{\text{Angular momentum}}{2 \pi \text{planet}} \]

We will discuss angular momentum later, but it is a vector with magnitude for unopposed circular motion: \( r \cdot v \) makes riding a bicycle possible!
Example: in Jack Van's novel

- Space ship

To earth

Close to earth → To moon

"up"

What is wrong?
tides

Tidal force results from $\Delta F_e$ over surface of Earth!

\[
\begin{align*}
\vec{F}_e - \Delta F_e & \\
\vec{F}_e + \Delta F_e & \\
\end{align*}
\]

Tidal force = $F_{\text{surface}} - F_{\text{moon}} = \Delta F$

Giving 2 tides/day

First calculated by Newton 1687

\[
\begin{align*}
M_e, M_m & \text{ mass of earth, moon} \\
R_e & \text{ radius of earth} \\
R_{E.m} & \text{ earth-moon distance}
\end{align*}
\]

difference in high-low ocean height

\[
\Delta = \frac{3}{2} \frac{M_m}{M_e} \frac{R_e^4}{R_{E.m}^3} \sin^2(90^\circ - \text{latitude})
\]

= 0.56 @ equator

Also, weather, inclination of Earth's axis, solar tide