E.O. Lawrence & the cyclotron

Cyclotron - a proton accelerator

Principle of operation: In B field, proton beam radius increases linearly with momentum, so that time to complete revolution is constant.

Direction of applied voltage varies with constant frequency. Works as long as $p = mv$.
\[ m \left( \frac{v^2}{c^2} \right) = q v B \Rightarrow m v = q B r \]

Frequency \[ f = \frac{v}{2 \pi r} = \frac{q B}{2 \pi m} \text{ constant} \]

But \( v < c \) while \( p \) continues to increase! Proton beam becomes out of sync with cyclotron frequency. Limits proton acceleration by cyclotron to \( K E \approx 6 \text{ MeV} \)

\[ p = m \frac{v}{\sqrt{1 - (v/c)^2}} \quad \text{"\( \gamma \)-factor"} \quad \sqrt{1 - (v/c)^2} \]
(2) Relativistic Energy momentum

\[ p = \gamma m v \]

Gamma factor \( \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \)

\[ E = \gamma mc^2 \Rightarrow \frac{v}{c} = \frac{pc}{E} \]

Rest energy \( E_v = mc^2 \)

These formulas hold for particles with mass. More generally,

\[ E^2 = (pc)^2 + (mc^2)^2 \]

So for massless particles, like photons

\[ E_{\text{photon}} = P_{\text{photon}} \cdot c \]

\[ \frac{v}{c} = \frac{pc}{E} = 1 \quad \text{massless particle moves at } v = c. \]
Einstein's theory

There is no medium ("aether") in which light propagates.

1. Principle of relativity - law of physics the same in all inertial frames.

2. Light travels in vacuum at speed c relative to all observers.

Michelson-Morley null experiment

An interferometer design to test for aether wind.

Light from source S

Path difference $M_1 - P$, $M_2 - P$, $P - R$ all the same = $\ell$.

* Non-accelerating with respect to "fixed" stars.
Math

path 1: \( p \to m_1 \to p \to R \)

\[ t_1 = \frac{3\beta}{c} \]

path 2: \( p - m_2 \to p - R \)

\[ t_2 = \frac{\beta}{c - v \gamma} + \frac{\gamma}{c + v \gamma} + \frac{\gamma}{c} \]

\[ t_2 - t_1 = \frac{2\beta}{c} \left[ \frac{(v \gamma / c)^2}{1 - (v \gamma / c)^2} \right] \]

measured \( \Delta t = 0 \).
Frames of reference:

\[ \begin{align*}
\Delta t & \quad \Delta \xi = 0 \\
A & \quad O
\end{align*} \]

Path of ball as viewed by B:

\[ v \Delta t = \Delta \xi' \]
Time dilation

\[ h = c \Delta t \]

\[ \Delta x' = v \Delta t' \]

\[ (c \Delta t')^2 = h^2 + (\Delta x')^2 = h^2 + (v \Delta t')^2 \]

\[ (c \Delta t')^2 - (v \Delta t')^2 = h^2 = (c \Delta t)^2 \]

\[ \Delta t' = \frac{\Delta t}{\sqrt{1 - (v/c)^2}} = \gamma \Delta t \]

\[ \Delta t' = \gamma \Delta t \]

Moving clocks run slow
Cosmic Ray muon

Muon lifetime: 2 μs

Muon mass = \( \frac{1}{16} m_p = 0.1 \text{ GeV} / c^2 \)

100 GeV muon has a \( \gamma \)-factor

\[
\gamma = \frac{E}{mc^2} = \frac{100 \text{ GeV}}{10 \text{ GeV}} = 10^3
\]

Traveling at "nearly the speed of light"

For \( \gamma \gg 1 \), \( V \approx c (1 - \gamma^{-2}) \)

\[
= 0.999999 \text{ c}
\]

Travels average distance

\[
d = c (\gamma \times \text{ lifetime}) = \\
\gamma (3 \times 10^8 \text{ m/s} \times 2 \times 10^{-6}) = 3.6 \text{ km}
\]
Relativity requires Lorentz contraction.

In muon rest frame, time interval $\Delta t = 2 \mu s$. Earth observer therefore only moves a distance $\nu \Delta t = 600 \text{ km} = d/y$.

![Diagram showing A views B measuring $\Delta x$ with Lorentz-contracted meter sticks.](image)