Beyond II

I. Grand Unification (GUT)

1. Two hints in Standard Model (SM):
   a. Running coupling (see previous lecture)
   b. Anomaly cancellation.

An anomaly is a quantum correction that breaks a symmetry of the classical Lagrangian. SM "triangle" anomaly term cancel identically if

\[ \sum Q_f = 0 \quad ; \quad Q_f = \frac{1}{3} + \frac{Y}{2} \]

for fermions

\[ Q_e + Q_u + Q_u + Q_d = -1 + 0 + 3 \left( \frac{2}{3} - \frac{1}{3} \right) = 0 \]

3 colors.

But SM gauge structure has no connection between \( SU_3(3) \) and \( SU_2(2) \times U_1(1) \).
Gauge Bosons

\[ \left[ SU(3) \times SU(2) \right]^Y \]

\[
\begin{align*}
(8, 1)_0 & \quad (1, 3)_0 & \quad (1, 1)_0 & \quad G_{\text{bino}}(G) \\
(1, 3)_0 & \quad (1, 1)_0 & \quad (3, 2)^{-5/3} & \quad W^+, W^0, W^- \\
(\bar{3}, 2)^{5/3} & \quad (\bar{3}, 2)^{5/3} & \quad X^{-4/3}, Y^{-1/3} & \quad \{ \text{Vector leptogluons} \}
\end{align*}
\]

Counting fermions: L, R state separately, SM

Left (left-handed):

\[ L: \nu, e_L, 3(U_L, d_L) \]

15 states

Right (right-handed):

\[ R: e_R, 3x(U_R, d_R) \]

Thus state fit into \( SU(5) \) multiplets.

\[ \begin{pmatrix} U_L \cr \bar{e}_R \cr \bar{\nu}_R \cr \bar{d}_R \cr \bar{u}_R \end{pmatrix} \quad L \rightarrow \begin{pmatrix} 0 & -\frac{1}{3} & -\frac{2}{3} & \bar{u}_R & \bar{d}_R \cr 0 & 0 & 0 & \bar{u}_R & \bar{d}_R \cr 0 & 0 & 0 & 0 & \bar{e}_R \cr 0 & 0 & 0 & 0 & 0 \end{pmatrix} \]

Where

(\( CP \) - invariant):

\( \bar{e}_R \) transforms like \( e_L \) (positive)

(\( CP \) - invariant):

\( \bar{e}_R \) transforms like \( e_L \) (electron)
Note: $\sum Q_f = 0$ for each multiplet ensures EM charge conservation.

Weinberg angle: determined by $SU(5)$ group theory or SM embedding.

\[ \sum \left[ Z \times \mathcal{F} \sin \theta \right] = 0 \]

For multiplet $I_3 = Q \sin^2 \theta_W$

\[ \text{giving} \quad \sin^2 \theta_W = \frac{\sum Q I_3}{\sum Q^2} \]

For $\frac{\sqrt{3}}{2}, \sqrt{3}$ gives $\sin^2 \theta_W = \frac{3}{8}$

Proton Decay:

$X_1, Y$ bosons violate baryon #, lepton #

(Preserve $B-L$) and mediate proton decay.

\[ e^- \rightarrow X^{-4/3} \]

\[ \int \frac{dX^{-4/3}}{X^{-4/3}} \]
so for example \( p \to e^+ \pi^0 \)

\[
\begin{align*}
\text{Current limit: } & \quad \mathcal{B}(p \to e^+ \pi^0) > 1.6 \times 10^{-3} \\
& \text{Super-K will continue to search.}
\end{align*}
\]

Other GUT Models: \( SO(10) \)

See fermions fit into some representation (multiplet) (plus 2\( \nu \)).
Supersymmetry:

Only possible extension of space-time symmetry (boosts, rotations, translation)

Fermi \leftrightarrow boson symmetry

every particle has susy partner (example):

spin \frac{1}{2} \ Fermi

\psi_L, \psi_R \ selection

\bar{\psi}_L, \bar{\psi}_R \ squarks

spin \ 0 \ Higgs

\tilde{h}, \ higgsero

spin \ -1 \ Boson

\tilde{W}, \ W \ gauginos

Higgs potential not put in by hand
Correction to MH cancel

Vacuum energy \ \Sigma \ zero

masses of particles = susy partners (coope!)
If SUSY is asymmetric of nature, then it is broken.

"Soft" breaking terms can be enumerated that allow TeV-scale SUSY breaking and still give (log divergent) corrections to Higgs mass.

Minimal field content MSSM

$\sim 120$ free parameters.

Additional symmetry $R$-parity prevents proton unacceptable proton decay.

Request SUSY particles to be pair produced.

Also implies lightest particle (e.g., $\tilde{g}$) is stable. Dark Matter candidate.