Lecture #14  Higgs Mechanism

Gauge Group \( SU_L(2) \times U_Y(1) \)

\[
Q = I_3 + \frac{Y}{2}
\]

Mix \( \rightarrow Z^0, A \)

\[
\frac{g'}{g} = \tan \theta_w
\]

\[
e = \frac{g g'}{\sqrt{g^2 + g'^2}}
\]

\( SU_L(2) \times U_Y(1) \rightarrow U_{em}(1) \)

**Spontaneous Symmetry Breaking (SSB)**

Classical analog - Compound rod

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<th>F</th>
<th>SSB</th>
<th>Bowed rod</th>
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H is rotationally invariant about z-axis, but ground state is not symmetric - special angle \( \theta \)

Rotations about \( \phi \) require no energy - / massless Goldstone boson for each broken symmetry.

Radial oscillations have large energy (mass).

Analog of the Higgs Boson.
Some simple examples (from Quigg, Gauge Theory)

Example 1: Discrete symmetry

$\phi(x)$ scalar field

$L = \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} \mu^2 \phi^2$

$\rightarrow$ eq. of motion, $\partial^2 \phi + \mu^2 \phi = 0$ 
Klein-Gordon free, massive scalar field, $\partial^2 = \partial_\mu \partial^\mu = (\frac{\partial^2}{\partial x^2} - \nabla^2)$

Add self interaction:

$V(\phi) = \frac{1}{4} \lambda \phi^4$

$L$ is invariant under $\phi \rightarrow -\phi$

$V(\phi) = \frac{1}{2} \mu^2 \phi^2 + \frac{2}{4} \phi^4$

Suppose that $\mu^2 < 0$. Then $V(\phi)$ has a minimum
\[ \frac{dV}{d\phi} = m^2 \phi + 2 \phi^3 = \phi \left[ -\frac{m^2}{2} + 2 \phi^2 \right] = 0 \]

\[ \nu = \pm \sqrt{-\frac{m^2}{\lambda}} \]

Ground state will be either \( \langle \phi \rangle = \pm \nu \)

breaking parity.

Oscillations about minimum, \( \phi' = \phi - \nu \)

\[ L = \frac{1}{2} \left( \partial_\mu \phi' \right)^2 + \frac{1}{2} \lambda \nu^2 (\phi' + \nu)^2 - \frac{\lambda}{4} (\phi' + \nu)^4 \]

Expanding and ignoring irrelevant constant terms:

\[ L = \frac{1}{2} \left( \partial_\mu \phi' \right)^2 - 2 \nu^2 \phi' \phi^{*} - 2 \nu \phi' \phi^3 - \frac{\lambda}{4} \phi' \phi^4 \]

\& mass term with correct sign
Example 2: breaking of continuous symmetry and Goldstone boson

\[ \phi = (\phi_1 + i\phi_2) / \sqrt{2} \]

is a complex scalar field described by a charged scalar particle.

\[ \phi^* \phi = \frac{1}{2} (\phi_1^2 + \phi_2^2) \]

The Lagrangian is invariant under global (constant) phase symmetry \( \phi \rightarrow e^{i\alpha} \phi \)

For \( \mu^2 = -2V^2 < 0 \), \( V \) has a minimum for \( V^2 = \phi_1^2 + \phi_2^2 \).

Choose \( \langle \phi \rangle = \frac{V}{\sqrt{2}} \) and expand about the minimum of "Mexican hat" potential:

\[ \Delta V(\phi) = -2V^2|\phi|^2 + 2^2|\phi|^4 \]
\[ \phi(x) = \frac{1}{\sqrt{2}} \left( \sigma(x) + \eta(x) + i \xi(x) \right) \]

\[ L = \frac{1}{2} (\partial \eta)^2 + \frac{1}{2} (\partial \xi)^2 + \kappa^2 v^2 \left[ (\sigma + \eta)^2 + \xi^2 \right] - \lambda \left[ (\sigma + \eta)^2 + \xi^2 \right]^2 \]

\[ = \frac{1}{2} (\partial \eta)^2 + \frac{1}{2} (\partial \xi)^2 - v^2 \kappa n^2 + \text{quartic + const} \]

\[ n \rightarrow \text{massive boson} \]

\[ \xi \rightarrow \text{massless (Goldstone) boson} \]
Standard Model

Simplest choice is \( Y = 1 \) weak iso-doublet

\[
\Phi = \begin{pmatrix} \Phi^+ \\ \Phi_0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \Phi_1 + i \Phi_2 \\ \Phi_3 + i \Phi_4 \end{pmatrix}
\]

2 complex (4 real) scalar fields.

\[
V(\phi) = \frac{1}{2} m^2 \Phi^\dagger \Phi + \frac{1}{4} \lambda (\Phi^\dagger \Phi)^2
\]

\[
\Phi^\dagger \Phi = -\frac{m^2}{2} = \left( \frac{v}{\sqrt{2}} \right)^2
\]

Choose broken direction \( \langle \Phi_3 \rangle = \sqrt{\frac{-m^2}{\lambda}} = v \)

\[
\langle \text{vac} | \Phi | \text{vac} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}
\]

Note that since \( \Phi = \frac{\Phi_3}{2} + \frac{\Phi_4}{2} \)

\[
\langle \hat{\Phi} \rangle = -\frac{1}{2} + \frac{1}{2} = 0
\]

\( \Phi \) is unbroken generator: EM charge is conserved

and photon is massless

\( \Phi_1, \Phi_2, \Phi_4 \) are massless Goldstone bosons

\( \Phi_3 \) massive Higgs
To make symmetry clean:

$$\Phi(x) = e^{-i \Theta(x)} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + h(x) \end{pmatrix}$$

A vi the physical Higgs particle

$\theta^+, \theta^-, \theta^0$ become longitudinal polarization

state of $\text{massive } W^+, W^-, Z^0$

$$\left| \left( g \vec{\tau} \cdot \vec{W}_\mu + \frac{g'}{2} \vec{B}_\mu \right) \phi \right|^2$$

$$= \frac{1}{8} \left| \begin{pmatrix} g w^3 + g' B \\ g w^- \\ g w^+ \\ -g w^3 - g' B \end{pmatrix} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \right|^2$$

$$= \left( \frac{1}{2} \nu \phi \right)^2 W^+ W^-$$

$$+ \frac{1}{8} \nu^2 (w^3, B) \begin{bmatrix} g^2 & g g' \\ -g g' & g'^2 \end{bmatrix} \begin{pmatrix} w^3 \\ B \end{pmatrix}$$

$$= m_{w}^2 W^+ W^- + \frac{1}{8} \nu^2 (g w^3 - g' B)^2$$

$$= m_{w}^2 W^+ W^- + m_{Z}^2 Z^2$$

where $m_{w} = \frac{1}{2} \nu g'$

$$m_{Z} = \frac{1}{2} \nu \sqrt{g^2 g'^2}$$
\[
\frac{M_W}{M_Z} = \cos \theta_W
\]

Orthogonal combination (photon) remains massless,

\[
A^\mu = \frac{g' W^3 + gB}{\sqrt{g^2 + g'^2}}
\]

\[
\frac{G_E}{\sqrt{2}} = \frac{g^2}{8 W W^2} = \frac{1}{2 v^2} \quad \Rightarrow \quad v = 286 \text{ GeV}
\]

\[
M_W = \frac{37.3 \text{ GeV}}{\mu \text{ rad}} ; \quad M_Z = \frac{79.6 \text{ GeV}}{5 \times 2 \text{ rad}}
\]

\[
M_H = \sqrt{2 v^2 \lambda} \quad \lambda \text{ is fine parameter}
\]
CKM matrix

Gr, Gv introduced to give Fermion masses are matrices in flavor space with the introduction of 3 families.

Diagonalize masses (Yukawa-Higgs couplings)

by rotating states (i = family index)

\[ U_{\nu_i}^{*} = U_{\nu_{i}\nu_{e}} U_{\nu_{e}} \]

where \( U_{\nu_{i}} \) are 3x3 arbitrary matrices, \( \nu \) index denotes diagonalized (mass) eigenstate.

This gives for the charged current:

\[ j^{\mu} = \frac{g}{\sqrt{2}} \, U_{\nu_{i}}^{*} \nu_{i} \, d_{L}^{\mu} = \frac{g}{\sqrt{2}} \, U_{\nu_{i}}^{*} \nu_{i} \, (U_{\nu_{e}}^{*} U_{\nu_{e}}) d_{L}^{\mu} \]

where \( U_{\nu_{i}} \) and \( d_{L}^{\mu} \) are the charged current.

\[ V_{ij} \times 3 \times 3 \ CKM \ matrix: \]

\[ \begin{pmatrix} d^{\nu i} \\ s^{\nu i} \\ b^{\nu i} \end{pmatrix} = \begin{bmatrix} V \end{bmatrix} \begin{pmatrix} d^{i} \\ s^{i} \\ b^{i} \end{pmatrix} \]

where a prime superscript means states that are \( \nu \) to the weak current.
Fermion masses

Also violate $SU_L(2)$ symmetry, fermions get mass from coupling to Higgs:

$$\Delta L^f = -G_f \left[ (\bar{\nu} \gamma^\mu \nu)_{L} \left( \phi^+ \right)_{R} + \bar{e}_R (\phi^-, \phi^0) (e)_L \right]$$

A free parameter

$$\Delta L_{SS8} = - \left( \frac{6e v}{\sqrt{2}} \right) \bar{e} e - \frac{G_f v}{\sqrt{2}} \bar{e} e \lambda$$

$$= -m_e \bar{e} e - \left( \frac{6e v}{\sqrt{2}} \right) \bar{e} e \lambda$$

Higgs coupling & mass

To get up quark masses, need charge-conjugate field

$$\phi^c = - \sqrt{2} \phi^+ = \sqrt{\frac{1}{2}} \left( \begin{array}{c} \nu+l_1 \\ 0 \end{array} \right), \quad Y = -1$$

$$\Delta L = -G_u \left( \bar{u} \gamma^\mu d \right)_L \left( \phi^+ \right)_R - G_d \left( \bar{d} \gamma^\mu s \right)_L \left( \phi^- \right)_R$$

$$\quad$$
This gives flavor changing charged current:

\[
\begin{bmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{bmatrix}
\]

3x3 Unitarity \Rightarrow 3 angles + a phase which are five parameters. Phase gives CP violation.

Angles are generalized Cabibbo angle

Neutral currents have no (leading order)

Flavor change:

\[
\bar{\nu}_e = \bar{\nu}_m U^\dagger_{em} u \Rightarrow \bar{\nu}_e \rightarrow \bar{\nu}_m U^\dagger_{em} u
\]

\[
\bar{d} = \bar{d}_m U^\dagger_{dm} d \Rightarrow \bar{d} \rightarrow \bar{d}_m U^\dagger_{dm} d
\]
$SM$ parameters:

- $\alpha, g_F, M_Z$
- $\theta_W$
- $\lambda$
- $M_H$
- $1/2^+$ fermion masses (and neutrino masses)
- 3 CKM angles
- 1 $CP$ violating phase

Enormously predictive. Large number of constraints when including quantum corrections:

$$\sin \theta_W \propto \ln \left( \frac{M_H}{M_W} \right)$$