Lecture 10: Parton Model

Inelastic e-N scattering:

\[ E'(1, \theta) \]

\[ E(1, \theta) \]

\[ \theta \]

\[ \rightarrow z \]

hadron \( (E_h, P_h) \)

Hadron mass \( m_h = \sqrt{E_h^2 - P_h^2} \)

Recall that for elastic scattering,

\[ E' = E \left/ \left(1 + \frac{2E}{M \sin^2 \frac{\theta}{2}}\right)\right. \]

for inelastic scattering, \( E', \theta \) are independent variables.

\[ \frac{d\sigma}{dE'd\Omega} = \frac{1}{4E^2} \frac{\cos^2 \frac{\theta}{2}}{\sin^4 \frac{\theta}{2}} \left( W_2 + 2 W_1 \tan^2 \frac{\theta}{2} \right) \]

where \( W_2, W_1 \) are form factors.

Or changing to variables, \( \nu = E - E' \) energy transfer:

\[ Q^2 = -Q^2 = 4EE' \cos^2 \frac{\theta}{2} \]

\[ \frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi (\alpha \mu_0^2)}{Q^4} \left( \frac{E'}{E} \right) \cos \frac{\theta}{2} \left[ W_2 + 2W_1 \tan^2 \frac{\theta}{2} \right] \]
where form factors are functions of \( Q^2 \),

\[
\text{Bjorken-Scaling}
\]

for \( Q^2 \geq \left( \frac{\hbar c}{0.1 \text{ fm}} \right)^2 = (2 \text{ GeV})^2 \)

\( W_2, W_1 \) depend only on dimensionless, invariant ratio ("Bjorken-\( x \")

\[
X = \frac{Q^2}{2 \mu M}
\]

\[
W_2^2 = \frac{Q^2}{2 \mu M}
\]

\[
W_1 \sim \frac{Q^2}{2 \mu M}
\]

Feynman's parton model

\[
\text{Wavelength of virtual photon } \lambda = \sqrt{\frac{\hbar c}{Q^2}}
\]

At large \( Q^2 \), strong force (QCD) couples i.e. weak at quarks are essentially free.
$x$ is interpreted as momentum fraction of proton carried by quark. Inelastic scattering of proton is then described by elastic scattering of quarks.

\[ \overline{k}' = \overline{e}'(1, \overline{r}') \]

\[ \overline{F} = \overline{F}(1, \overline{r}') \]

\[ f_i(x) \text{ probability for quark of flavor } i \text{ to have momentum fraction } x. \]

\[ (x \overline{p} + \overline{q})^2 = m_i^2 \geq 0 \text{ neglect quark mass.} \]

\[ \overline{q}^2 + 2 \cdot \overline{p} \cdot \overline{q} + x^2 \overline{p}^2 = 0 \]

\[ x = \frac{Q^2}{2 \overline{p} \cdot \overline{q}} \text{ } \overline{\text{lab.}} = \frac{Q^2}{2M} \]
Compare elastic $e-\mu$ scattering to inelastic $e-p$:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi}{Q^2} (\frac{E}{E'})^2 \cos^2 \frac{E}{2} \left[ 1 + \frac{Q^2}{2m^2} \frac{m_\mu^2}{m^2} \right]$$

When the limit with the identification of $W_1^{\text{in}}$, $W_2^{\text{in}}$ are:

$$W_1^{\text{in}} = \frac{Q^2}{4m^2} \delta \left( \nu - \frac{Q^2}{2m} \right)$$

$$W_2^{\text{in}} = \delta \left( \nu - \frac{Q^2}{2m} \right)$$

where $\nu = \frac{Q^2}{2m}$ corresponds to $X = 1$

Note:

$$\delta \left( \nu - \frac{Q^2}{2m} \right) = \frac{\delta (E-E')}{E/E'}$$

where $E' = \frac{E}{1 + \frac{2E\mu m^2 \theta_1}{m}}$
Define covariant dimensionless variables

\[ y = \frac{\overline{p}}{p} = \frac{1}{\sin^2 \theta/2} \text{ zero-momentum} \]

\[ E - E' = \frac{y}{E} \text{ let from} \]

\[ S = E_{2m}^2 = (E + m, E)^2 = 2mE \]

\[ \mu \ll E \]

\[ x = Q^2/2m\nu = Q^2/sy \]

\[ \frac{dx}{dy} = \frac{4\pi (x + 2\nu)^2}{Q^4} \left\{ \frac{1}{2} \left[ \frac{1}{1 + (1-y)^2} \right] - \frac{mxy}{2E} \right\} \]

Direct scattering e- by pointlike Q=1 Fermi with

momentum x\overline{p}^x \]

\[ e - p \text{ scattering } \] define dimensionless structure functions \[ F_1 = NW_1, F_2 = VW_2 \]

\[ \frac{dx}{dy} = \frac{4\pi (x + 2\nu)^2}{Q^4} \left\{ \frac{1}{2} \left[ \frac{1}{1 + (1-y)^2} \right] 2xF_1 \right\} \]

\[ + (1-y) \left( F_2 - 2xF_1 \right) - \frac{m}{2E} xy F_2 \right\} \]

Comparing, we see that for parton model

\[ F_2 = 2x F_1 = \sum_i Q_i^2 x_i f_i(x) \]
\[
\frac{d\sigma}{dxdy} = \frac{4\pi (x+y)^2}{Q^2} \sum_x \left\{ \frac{1}{2} \left[ 1 + \frac{m^2}{2e} xy \right] - \frac{m}{2e} xy \right\} \frac{F_2}{x}
\]

\[
\frac{d\tau}{dxdy} = \frac{d\tau}{dy} \sum q_i^2 \delta_i(x)
\]

It was found that proton contains not only \(u,d\) "valence quarks," but also \(\bar{u}, \bar{d}\) pair "sea quarks."

\[\bar{u} \bar{u}\] pair from gluon exchange:

So \(F_0(x) \equiv U(x)\) contains valence and sea quarks.
proton structure function:

\[ F_2^p(x) = \frac{4}{9} x \left[ u(x) + \bar{u}(x) \right] \]

\[ + \frac{1}{3} x \left[ d(x) + \bar{d}(x) + s(x) + \bar{s}(x) \right] \]

with sum rule:

\[ \int_0^1 dx \ x \ (u + \bar{u} + d + \bar{d} + s + \bar{s}) = 1 \]

\[ \int_0^1 dx \ [u - \bar{u}] = 2 \]

\[ \int_0^1 dx \ [d - \bar{d}] = 1 \]

\[ \int_0^1 dx \ [s - \bar{s}] = 0 \]

\[ F_2^p(x) \] agree for e-\(p\), \(\mu\)-\(p\), \(\nu\)-\(p\) scattering. However:

\[ \int_0^1 dx \ x \left[ u + \bar{u} + d + \bar{d} + s + \bar{s} \right] \approx \frac{1}{2} \]

the rest is glue!
Electron scattering

Scattering of point (elementary, structureless) particles provides easy to interpret experiments.

**Bhabha scattering**: \( e^+ e^- \rightarrow e^+ e^- \)

\[
\frac{d\sigma}{d\Omega} = \frac{(k\pi)^2}{8E^2} \left[ \frac{1 + \alpha^2 s^2}{\sin^2 \theta} + \frac{1}{2} (\cos^2 \theta + 2 \sin^2 \theta) \right]
\]

Ultra-relativistic limit:

\[
|A|^2 \propto \frac{s}{4E} \frac{1}{s^2 + m^2}
\]

\( E \) and \( e^- \) energy in \( Z \) frame

\( \chi^2 \): \( t = s^2 - 4E^2 \sin^2 \theta \)

\( S \): \( s = (E(1,2) + E(1,-2))^2 = 4E^2 \)

\( t, s \) are square of 4-momenta of virtual exchange photons \( t \) channel amplitude diverges as \( 0^{-+} \) just like Rutherford.
\[ e^+ e^- \rightarrow \mu^+ \mu^- \]

\[ \frac{d\sigma}{d\Omega} = \frac{(\gamma + \kappa)^2}{4E^2} (1 + \kappa \cos \theta) \]

Can be understood in terms of angular momentum conservation.

\[ S \rightarrow \rho^+ \quad \text{helicity} = +1, \quad \text{"right handed"} \]

\[ \rho^- \rightarrow \bar{\rho} \quad \text{helicity} = -1, \quad \text{"left handed"} \]

At high energy (\( m = 0 \) limit) helicity is conserved by electromagnetic interaction.

Position-like electron mirror backwards in time.
Angular momentum conservation gives

\[ A(LR \rightarrow LR) \propto \alpha_{1,1}^{t} (\theta) = \frac{1 + \cos \theta}{2} \]

3 outer amplitudes:

\[ A(LR \rightarrow RL) \propto \alpha_{1,-1}^{t} (\theta) = \frac{1 - \cos \theta}{2} \]

\[ A(RL \rightarrow RL) \propto \frac{1 + \cos \theta}{2} \]

\[ A(RL \rightarrow LR) \propto \frac{1 - \cos \theta}{2} \]

All amplitudes are distinguishable, so squaring and summing gives

\[ P(\theta) = \frac{1}{2} (1 + \cos \theta) \]

\[ e^+e^- \rightarrow 2 \text{jet} \] angular distribution of jet axis has \( \frac{1 + \cos \theta}{2} \) distribution \( \Rightarrow \) quarks have spin \( \frac{1}{2} \).