HW # 3
Due Wednesday, Feb 14, 2007

#1) Define $T_{\text{max}}$ as the maximum kinetic energy which can be imparted to a free electron in a single collision. Show that for a particle of mass $M$, velocity $\beta c$ and momentum $\beta \gamma M c$

$$T_{\text{max}} = \frac{2m_e c^2 \gamma^2 \beta^2}{1 + (m_e/M)^2 + 2\gamma m_e/M}$$

#2) from Perkins
The average number $\bar{n}$ of ionizing collisions suffered by a fast particle of charge $ze$ traversing an interval $d\chi$ ($\text{g cm}^{-2}$) of a medium, and resulting in energy transfers $E \rightarrow E + dE$ is

$$\bar{n} = f(E)dE d\chi = K \frac{Z m_e c^2 dE}{A \beta^2 E^2 (1 - \beta^2 E/E_{\text{max}}^2)} d\chi,$$

where $K = 0.3 \text{cm}^2/\text{mole}$ and $E_{\text{max}} = 2(m_e c^2)\beta^2 \gamma^2$ (from #1 above). For individual particles, the distribution in number of collisions $n$ obeys Poisson statistics, so that $\langle (n - \bar{n})^2 \rangle = \bar{n}$. If we multiply the above equation by $E^2$ and integrate, we obtain the mean square deviation in energy loss $\sigma^2 = \langle (\Delta E - \bar{\Delta E})^2 \rangle$. Show that

$$\sigma^2 = 0.6 \frac{Z}{A} z^2 (m_e c^2)^2 \gamma^2 (1 - \beta^2/2) \Delta \chi$$

Calculate the fractional rms deviation $\sigma/\Delta \chi$ in energy loss for protons of kinetic energy 500 MeV traversing a thickness (a) 0.1, (b) 1.0 and (c) 10 g/cm$^2$ of plastic scintillator $(Z/A = 1/2)$. Take $dE/d\chi = 3 \text{MeV/(g cm}^2)$. See #2) above. $(m_p = 938, m_\pi = 140, m_k = 494 \text{MeV/c}^2)$

#3) from Perkins
It is sometimes possible to differentiate between the tracks due to relativistic pions, protons, and kaons in a bubble chamber by virtue of the high energy $\delta$-rays which are produced. For a pion of momentum 5 GeV/c, what is the minimum energy of a $\delta$-ray which must be observed to prove it is not produced by a kaon or proton? What is the probability of observing such a knock-on electron in 1 m of liquid hydrogen (density = 0.06 g/cm$^3$)? See #2) above. $(m_p = 938, m_\pi = 140, m_k = 494 \text{MeV/c}^2)$