#1) Proof of the existence of neutral currents was established by neutrino beam experiments. Show that the process $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$ proves the existence of neutral currents, whereas $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ does not. Show that the maximum angle of emission of the recoil electron (initially at rest in the Lab) relative to the neutrino beam direction is $\sqrt{2m/E}$, where $m, E$ are the electron mass and final energy.

#2) The neutral current is:

$$j_{NC} = \frac{g}{c_W} \left[ j_3^L - s_W^2 j_{EM} \right]$$

Show that it can be re-written as:

$$j_{NC}^\mu = \frac{g}{2c_W} \bar{\psi} \gamma^\mu (g_V - g_A \gamma^5) \psi$$

or

$$j_{NC}^\mu = \frac{g}{c_W} \left[ g_L \bar{\psi}_L \gamma^\mu \psi_L + g_R \bar{\psi}_R \gamma^\mu \psi_R \right]$$

where $g_V = g_L + g_R$, $g_A = g_L - g_R$. Express $g_V, g_A$ in terms of the third component of the weak isospin $I_3^{weak}$, the fermion charge in units of $e$ ($q$), and the Weinberg angle ($\sin \theta_W$).

#3) Make a table of the couplings $g_L$, $g_R$ for a generation of the Standard Model fermions.

#4) The partial width of the Z to a fermion-antifermion pair is given by:

$$\Gamma(Z \rightarrow \bar{f}f) = \frac{G_F M_Z^3}{6\pi \sqrt{2}} \left( g_V^2 + g_A^2 \right)$$

Find the branching ratios:

a) leptonic: $\Gamma(Z \rightarrow \bar{\ell}\ell)$, where $\ell = e, \mu, \tau$

b) hadronic: $\Gamma(Z \rightarrow \bar{q}q$, the hadronic decay mode (don’t forget color!)

c) invisible: $\Gamma(Z \rightarrow \bar{\nu}\nu$, where $\nu$ is any flavor of neutrino.