Physics 450/II Final

Due Friday May 13. You may use class notes, problem set solutions, our textbook, and Clebsh-Gordon tables

#1) A $K^0$ mesons decays isotropically in its rest frame into $\pi^+\pi^-$. The mass of the $K^0 = 0.498$ GeV, and the mass of the charged pion is 0.140 GeV. In the laboratory, the $K^0$ momentum is $\vec{k}$. In terms of $k$ and the charged pion momentum $p^*$ in the kaon rest frame, calculate and sketch the distributions in the lab frame of:
   a) $dN/dE$ for the pion (lab) energy $E$;
   b) $dN/dp_\perp$ for the pion (lab) momentum component $p_\perp$ perpendicular to the kaon momentum $\vec{k}$.

#2) Consider the ratio of decay rates,
   \[ \frac{\Gamma(\Sigma^+ \rightarrow \pi^0 p)}{\Gamma(\Sigma^+ \rightarrow \pi^+ n)} \]
   a) If this were a strong decay, the final state would have to have definite isospin. What are the possible isospin values? For each possibility, what would you predict for this ratio. (Look up Clebsh-Gordon coefficients.)
   b) Draw the (leading order) Feynman diagrams for these decays in the Standard Model. What does the Standard Model predict for the ratio. Recall,
      \[ d' = d \cos \theta_C + s \sin \theta_C, \]
      \[ s' = -d \sin \theta_C + s \cos \theta_C. \]

#3) From the fact that the isotopes $^4H$, $^4Li$ do not exist, what do you conclude about the isospin of $^4He$? Do you expect a bound state of 4 neutrons to exist? Why or why not?

#4) The neutral-current (NC) and charged current (CC) neutrino-nucleon cross-sections are given by,
   \[ \frac{d\sigma^{\nu N}_{\text{NC}}}{dy} = A \sum_i \left[ (g^i_L)^2 + (g^i_R)^2(1 - y)^2 \right]; \quad \frac{d\sigma^{\nu N}_{\text{CC}}}{dy} = A \]
   whereas the antineutrino-nucleon cross-sections are given by,
   \[ \frac{d\sigma^{\bar{\nu} N}_{\text{NC}}}{dy} = A \sum_i \left[ (g^i_L)^2(1 - y)^2 + (g^i_R)^2 \right]; \quad \frac{d\sigma^{\bar{\nu} N}_{\text{CC}}}{dy} = \frac{A}{3}. \]
   Here $A$ is a constant, the sums are over $i = u, d$ quarks, and the kinematic variable $y$ has range $0 < y < 1$ (incidentally, it is the square of the sin of half the CM scattering angle). Calculate the cross-section ratios $\sigma^{\nu N}_{\text{NC}}/\sigma^{\nu N}_{\text{CC}}$, $\sigma^{\bar{\nu} N}_{\text{NC}}/\sigma^{\bar{\nu} N}_{\text{CC}}$ in terms of $\sin^2 \theta_W$. Recall that: $g_L = I^3 - Q \sin^2 \theta_W$, $g_R = -Q \sin^2 \theta_W$, where $Q$ is the fermion charge in units of the electron charge $|e|$. Another non-trivial prediction of the Standard Model!