Lecture #1: What is quantum mechanics?

I. Mechanics vs. Scarcity of motion

Classical particle: $\vec{r}(t), \vec{p}(t)$

Equation of motion: $\vec{F} = \frac{d\vec{p}}{dt}$, can be solved to give trajectory given $\vec{F}(0), \vec{p}(0)$

$$\begin{align*}
\frac{d^2 z}{dt^2} &= -mg \\
z(t) &= z_0 + V_0t - \frac{1}{2}gt^2
\end{align*}$$

Trajectory $z(t)$ for constant acceleration

"Atomic scale" particle (e.g. $e^-, p$) do not have trajectories. They propagate as waves with de Broglie wave lengths:

$$\lambda = \frac{h}{p} \quad \text{(holds relativistically)}$$

Wave propagation exhibits interference and diffraction.

Davidson & Germer observed diffraction of $e^-$ by Ni crystal.
Wave-particle duality:
- E.m. particles propagate as waves; interact as particles.

Q. What determines whether a particle is classical or quantum?
Can we be more precise than "atomic scale"?

III. Wave propagation - interference

\[ \sin(\Theta) \approx \Theta \]

Slits \( s_1, s_2 \) act like point sources of spherical waves that are in phase.

Interference of wave amplitudes results from path difference \( \Delta \).

\[
\Delta = \sin \Theta d \approx \Theta d = \frac{\theta}{1} d \quad \text{if } y \gg d
\]

\[
\Delta = \frac{\pi y}{d} \Rightarrow \begin{cases} n\lambda & \text{constructive} \\ (n+\frac{1}{2})\lambda & \text{destructive} \end{cases}
\]
Wave amplitude (plane wave)

\[ \varepsilon(x,t) = \varepsilon_0 \cos \left( \frac{kx - \omega t + \phi}{\lambda} \right) \]

\( \phi = \text{phase} \)

\( \omega = 2\pi f, \quad k = \frac{2\pi}{\lambda} \)

Phase velocity \( d\phi = kd\chi - \omega dt = 0 \)

\[ V_{\text{phase}} = \frac{dx}{dt} = \frac{\omega}{k} = V \]

Interference is the result of superposition:

\[ \varepsilon_1 + \varepsilon_2 = \varepsilon_0 \cos \left( kx - \omega t \right) + \varepsilon_0 \cos \left( k(x+\alpha) - \omega t \right) \]

Convenient to use complex amplitudes and take real part in the end.

In q.m., complex amplitudes are necessary.
Complex numbers

\[ x^2 + 1 = 0 \quad \text{solutions} \quad \sqrt{-1} = i \]

A complex number \( z = a + bi \)
- Real part
- Imaginary part

\[ z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2) \]

\[ z_1 \cdot z_2 = (a_1 + ib_1)(a_2 + ib_2) = a_1a_2 - b_1b_2 + i(a_1b_2 + a_2b_1) \]

Complex plane

\[ |z| = \sqrt{a^2 + b^2} = \sqrt{zz^*} \]

When complex conjugate \( z^* = a - ib \)

Euler identity \( e^{i\theta} = \cos \theta + i \sin \theta \) \((e^{i\pi} = -1)\)

\[ \cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}; \quad \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i} \]

And \( z = |z|e^{i\phi} \quad \phi = \tan^{-1}\left(\frac{b}{a}\right) \)
1/20/05

due Monday following

Ch 5: 2, 11, 18, 23, 31, 34

Electron current: talk
Wave packets duality - prob. amplitude
Size of atom
n decay
Lifetime
Wave packet
1. Sanders & Berlin
2. Friday feature - Fullenweir
3. AXAP and wave packets, atomic size
4. constant wave packet

Hessberg's microscope + decoupler

source neutra
\[ \varepsilon_1 + \varepsilon_2 = \varepsilon_0 e^{ikx-\omega t} \left( 1 + e^{ik\Delta x} \right) \]
\[ = \varepsilon_0 e^{ikx-\omega t} \left( e^{\frac{ik\Delta x}{2}} - e^{-\frac{ik\Delta x}{2}} \right) \]
\[ I = |\varepsilon_1 + \varepsilon_2|^2 = 4\varepsilon_0^2 \varepsilon_c^2 \left( \frac{k \Delta x}{2} \right) \]
\[
\frac{k \Delta x}{2} = \frac{2\pi}{\lambda} \frac{\Delta x}{2} = \pi \left( \frac{\Delta x}{\lambda} \right)
\]
\[ \Delta x = \begin{cases} n \pi \text{ constructive} & n = 0, 1, 2, \ldots \\ \left( \frac{2n+1}{2} \right) \pi \text{ destructive} & \end{cases} \]

**W. Davison - Germer**

Interference of the resulting summing and squaring amplitudes.

\[ d \]
\[ \Delta x' \]
\[ \frac{\lambda}{2} \]
\[ \frac{d \sin \theta}{2} \]
\[ \lambda = \frac{2d \sin \theta}{n} \]
\[ E_R = eV = E - mc^2 \]

\[ E = \sqrt{(pc)^2 + (mc^2)^2} = \gamma mc^2 \]

\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ \frac{E_R}{mc^2} = \gamma - 1 \ll 1 \quad \text{particle is non-relativistic} \]

\[ \gamma - 1 \ll 1 \Rightarrow 1 - \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \ll 1 \]

\[ \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \ll 1 \Rightarrow 1 + \frac{1}{2} \left(\frac{v}{c}\right)^2 - 1 \ll 1 \ll 1 \]

\[ E_R = mc^2 (\gamma - 1) \approx mc^2 \frac{1}{2} \left(\frac{v}{c}\right)^2 = \frac{1}{2} m v^2 = \frac{p^2}{2m} \]

and \( \beta = \frac{v}{c} \)

\[ mc^2 = 511 \text{ keV} \]

For \( V = 1 \text{ V}\), \( E_R = 1 \text{ eV} \), \( \lambda = \frac{h}{p} \)

\[ E_R = \left(\frac{h}{p}\right)^2 \frac{1}{2m} \Rightarrow \lambda = \frac{hc}{\sqrt{2mc^2 E_R}} \]

\[ h c = 1240 \text{ eV} \cdot \text{nm} = 1240 \text{ MeV} \cdot \text{fm} \]

\[ \lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2(1\text{ eV}) (511 \times 10^5 \text{ eV})}} = 1.23 \text{ nm} \]
Lecture #1

David Geniu

from X-ray diffraction from Ni crystal

first e- intensity peak measure

de Broglie $\lambda$:

$\lambda = 2d \sin \Theta / n$

measure $\lambda$ for various voltages

slope = $(1.23 \pm 0.0) nm (\text{Volt})^{1/2}$

Wave/particle duality?

$\lambda$ - diffraction envelope

$I$ = probability of detecting particle (e-) at position $y$

particle as detected, wave at points (interfer as particle)

Diffraction pattern emerge, statistically