Modern Physics 330: HW # 4

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problems from chapter six
#1)6.3 **#2)**6.4 **#3)**6.8 **#4)**6.57

#4) Barrier scattering

First we consider a particle incident on the barrier of height V_0 and width a with energy $E > V_0$. Set up the problem as in class: $k = \sqrt{2mE}/\hbar$, $q = \sqrt{2m(E-V_0)}/\hbar$ and the wave function in the three regions as:

$$\psi_1 = Ae^{ikx} + Be^{-ikx}, \quad x < 0$$
$$\psi_2 = C\cos qx + D\sin qx, \quad 0 < x < a$$
$$\psi_3 = Fe^{ikx}, \quad x > a$$

Apply the boundary conditions at $x = 0$ and $x = a$. Solve for the ratio F/A to get,

$$(F/A)^{-1} = \Big[\cos(qa) - i\sin(qa)\Big(\frac{k^2+q^2}{2kq}\Big)\Big]e^{ika}$$

Find $T^{-1} = |F/A|^{-2} = [(F/A)(F^\star/A^\star)]^{-1}$, where "$\star$" denotes complex-conjugation.

What happens when $qa = n\pi$?

#5) Tunneling

Use the formula for the *amplitude* from barrier scattering and the technique of "analytic continuation" to get the barrier penetration formula: let $q = i\alpha \equiv i\sqrt{2mV_0 - E}/\hbar$ in the formula for $(F/A)^{-1}$.

Take the "square" $|F/A|^2 = [(F/A)(F^\star/A^\star)]$ to get the tunneling formula given in the text, and derive the approximate exponetial form (see problem 6.61).

#6) The Momentum Operator

In this problem you will show that for a free particle,

$$m\frac{d\langle x\rangle}{dt} = \int dx\psi^\star\frac{\hbar\partial}{i\partial x}\psi$$

Bring the time derivative inside the integral $\langle x\rangle = \int dx\psi^\star x\psi$.
You should not take the derivative of the x in the integral! Why?

Use the Schrodinger equation for a free particle (and the complex-conjugate equation) to re-write the time derivatives on the wave function (and complex-conjugate). Use integration by parts and make a physical argument for throwing away the total derivatives which integrate to terms like $\frac{\partial \psi^{\star}}{\partial x} x \psi$ evaluated at $\pm\infty$