Modern Physics 330: HW # 1

#1) Lorentz Transformation By judiciously picking specific space-time points in a certain reference frame, show that moving clocks run slow, and that moving meter sticks are contracted. (Hints: Proper time is the time measured in the rest frame of the clock, Proper length is the length measured in the rest frame of the meter stick. A *length* in any frame is the distance between two positions measured at the same time.)

#2) Simultaneity

Given two events $\bar{X}_o \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\bar{X}_a \rightarrow \begin{pmatrix} \epsilon x \\ x \end{pmatrix}$,where $0 < \epsilon < 1$, find the boost to the frame in which these events are simultaneous.

#3) The Train Taggers Paradox

Two graffiti "taggers" are standing next to the train tracks separated by a distance ℓ as measured in their frame (frame S). As a train traveling at near light speed goes by, they simultaneously in their frame mark the train with paint. What is the proper distance Λ between the marks, that is the distance measured by observers aboard the train (frame S$'$)?

Show that consistent answers are obtained in both frames. Place one of the taggers at the origin $x = x' = 0$ and the other at $x = \ell$. Define the following events:

1. o: common spacetime origin $\bar{X}_o \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ in both frames.
2. e: $\bar{X}_e \rightarrow \begin{pmatrix} 0 \\ \ell \end{pmatrix}$ in frame S.
3. f: $\bar{X}_f \rightarrow \begin{pmatrix} 0 \\ \Lambda \end{pmatrix}$ in frame S$'$.

Determine the coordinates for these events in both frames. Describe how these events are observed in each frame.

#4) A proton with momentum 200 GeV/c in the laboratory collides with a proton at rest. Calculate the total energy in the center-of-momentum frame.

#5) A photon with energy equal to the electron rest energy ($= mc^2$, where m is the electron mass) collides with an electron at rest and scatters at an angle of $\pi/2$. What is the recoil energy of the electron (as a fraction of mc^2)?

#6) A photon of energy 10 eV collides with an electron that has an energy of 10 GeV. If the photon is back-scattered, what is the final energy of the photon and electron?

#7) What is the minimum photon energy required to produce can electron-positrion pair in a collision with an electron at rest?
$\gamma + e^- \to e^- + e^+ + e^-$ (Note, the mass of the positron is identical to that of the electron.)

#8) Prove that the photo-electic effect cannot work for a free electron, that is the process $\gamma + e^- \to e^-$ is forbidden by energy-momentum conservation.

#9) A pi-zero has a mass of $m_\pi c^2 = 135 MeV$ and decays into two photons. In the laboratory, the pi-zero is moving in the x-direction. After the decay, one photons travels in the x-direction, and the other in the minus-x direction. Calculate the laboratory momentum of the pi-zero as a function of the energy difference between the two observed photons.

#10) A pi-zero is produced in the laboratory with momentum of 1 GeV/c and moving in the x direction. It decays into two photons. In the rest frame of the pi-zero, the photons are produced back-to-back, one with an angle of $\theta^\star$ with respect to the x direction. Calculate the angle between the photons in the laboratory frame as a function of the decay angle in the pi-zero rest frame ($\theta^\star$). Note, do not assume that the photons will not always go in the same (or opposite) directions for all $\theta^\star$.