Modern Physics 330: Exam # 3 SOLUTIONS

Open notes, open textbook calculator ok. 1 hour.

#1) For the n=3 level of hydrogen:

a) How many electron states are there in total, including spin? $2 \cdot 3^2 = 18$

b) What is the energy eigenvalue E_3 in electron volts? $-13.6/9$ eV.

c) Write the multiplets of total $\vec{J} = \vec{L} + \vec{S}$ for $n = 3$ in the notation nL_j. Show that the total number of states summed over multiplets equals your result in part (a). $3S_{\frac{1}{2}}(2),\ 3P_{\frac{1}{2}}(2),\ 3P_{\frac{3}{2}}(4),\ 3D_{\frac{3}{2}}(4),\ 3D_{\frac{5}{2}}(6)$

d) Calculate the spin orbit correction for each multiplet in part (c) using,

$$E_{SO} = \frac{a}{\hbar^2}\langle 2\vec{L} \cdot \vec{S}\rangle = a\left[j(j+1) - \ell(\ell+1) - \frac{3}{4}\right]$$

where a is positive number with dimensions of energy.

$$3S_{\frac{1}{2}} = 0,\ 3P_{\frac{1}{2}} = -2a,\ 3P_{\frac{3}{2}} = +a,\ 3D_{\frac{3}{2}} = -3a,\ 3D_{\frac{5}{2}} = +2a$$

e) What is the physical reason that the multiplets remain degenerate as a result of this splitting? The total angular momentum is still conserved.

f) What is the order of magnitude you expect for the ratio E_{SO}/E_3 ? $E_{SO}/E_3 \propto \alpha^2 \sim 10^{-4}$

#2)

a) Using the diagram below, write the configuration for the ground state of the potassium K (Z=19). $1S^2 2S^2 2P^6 3S^2 3P^6 4S^1$

b) What is the physical reason that the lower ℓ-states get filled first? The lower ℓ states have Radial P.D.F.'s that are larger at lower r (smaller angular-momentum barrier) and therefore see less screening of the nuclear charge by other electrons.

#3) What are the electric-dipole allowed transitions for the hydrogen $3D_{\frac{3}{2}}$ state? $nP_{\frac{1}{2}}, nP_{\frac{3}{2}}$

#4) Positronium is an electron-positron bound state. It is almost identical to the hydrogen atom except that now the proton is replaced by a positron, and the positron's mass is equal to the electron's mass.

a) What is the ground state energy?

The reduced mass is now $m_e/2$, so $E = -\mu c^2\alpha^2/2 = -13.6/2$eV.

b) What would you expect for the hyperfine splitting relative to that in hydrogen? The magnetic moment is inversely proportional to the mass, so replacing the proton mass by the positron mass means the the HF splitting increases by a factor of $m_p/m_e \approx 2000$.