Modern Physics 330: Exam # 3

Open notes, open textbook calculator ok. 1 hour.

#1) A neutron is subject to the gravitational potential sketched in Figure 1.

\[ V(z) = mgz, \quad z > 0; \quad V(z) = \infty, \quad z < 0, \]

where \( g \) is the acceleration due to gravity at the surface of the earth.

a) On the figure, identify the classical turning point of the particle at \( z > 0 \). Divide the \( z > 0 \) axis into two regions according to whether the particle is classically allowed. Label these regions as oscillatory (“osc”) or real exponential (“exp”) according to the behavior you expect for the energy eigenstate wave functions. Sketch the ground state wave-function on the same \( z \)-axis as the potential.

b) The characteristic length of the problem is the combination of constants:

\[ \ell \equiv \left( \frac{\hbar^2}{m_N g} \right)^{\frac{1}{3}}. \]

Show that this combination has dimensions of a length. Note that \( m_N g \ell = \hbar^2 / (m_N \ell^2) \) is an energy.

c) Use the uncertainty principle to estimate the ground state energy. Express your answer in terms of \( \hbar, m_N \) and \( \ell \).

d) Use your estimate of the ground state energy to obtain a numerical value (number) for the classical turning point of part (a). Use the value \( \ell = 15 \) microns.
Figure 1: Potential of neutron in gravitational field. The potential is infinite for $z < 0$. 
Consider scattering off of a “step down potential” (Figure 2), with the particle incident from the negative x direction.

a) Write the general solution to the time-independent Schrödinger equation for the regions $x < 0$ and $x > 0$, taking into account the initial condition of the particle incident from the negative x direction.

b) Use the boundary conditions on the wave function to write linear equations for the unknown constants of your general solution of part a. What are the physical reasons for these boundary conditions?

c) Sketch the solution, using the same x-axis as in the figure.

d) The reflection coefficient is:

$$ R = \left( \frac{k' - k}{k' + k} \right)^2 $$

where $k$, $k'$ are the wave number for $x < 0$, $x > 0$ respectively. Take $E = (3/2)V_0$. What is the numerical value for R? (give a number)

e) What is the numerical value for T? (give a number)

f) What would you expect for a classical particle for R, T?

Figure 2: One dimensional scattering from a step-down potential: $V(x) = V_0$, $x < 0$ and $V(x) = 0$, $x > 0$. 
#3) Recall the quantum mechanical problem of a particle in box.
   a) In what sense does the correspondence principle hold in this example?
   b) Consider a superposition state,
      \[ \phi = A(\phi_1 + 2\phi_2), \]
      where \( \phi_n \) are the normalized energy eigenfunctions with eigenvalues \( E_n \) and A is a normalization constant. What is A for \( \phi \) to be properly normalized? Note that no explicit integrals are needed; use the orthogonality of the eigenstates, \( \int \phi_n^* \phi_m dx = \delta_{nm} \).

   c) What is the probability to measure \( E_1 \)? \( E_2 \)?
   d) What is the expectation value of the energy, expressed as a constant times \( E_1 \)?
   e) Repeat b,c,d for the state,
      \[ \phi = A(\phi_1 + 2i\phi_2). \]